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## **Basic Fundamentals of Gear Drives**

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## **BASIC FUNDAMENTALS OF GEAR DRIVES**

A gear is a toothed wheel that engages another toothed mechanism to change speed or the direction of transmitted motion. Gears are generally used for one of four different reasons:

1. To increase or decrease the speed of rotation;
2. To change the amount of force or torque;
3. To move rotational motion to a different axis (i.e. parallel, right angles, rotating, linear etc.); and
4. To reverse the direction of rotation.

Gears are compact, positive-engagement, power transmission elements capable of changing the amount of force or torque. Sports cars go fast (have speed) but cannot pull any weight. Big trucks can pull heavy loads (have power) but cannot go fast. Gears cause this.

Gears are generally selected and manufactured using standards established by American Gear Manufacturers Association (AGMA) and American National Standards Institute (ANSI).

This course provides an outline of gear fundamentals and is beneficial to readers who want to acquire knowledge about mechanics of gears. The course is divided into 6 sections:

- Section -1 Gear Types, Characteristics and Applications
- Section -2 Gears Fundamentals
- Section -3 Power Transmission Fundamentals
- Section -4 Gear Trains
- Section -5 Gear Failure and Reliability Analysis
- Section -6 How to Specify and Select Gear Drives

## SECTION -1            GEAR TYPES, CHARACTERISTICS & APPLICATIONS

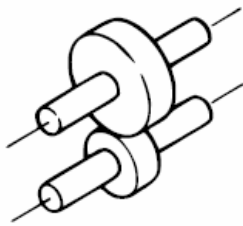
The gears can be classified according to:

1. the position of shaft axes
2. the peripheral velocity
3. the type of gears
4. the teeth position

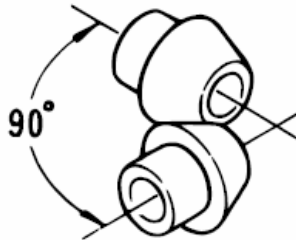
### According to the position of shaft axes:

Gears may be classified according to the relative position of the axes of revolution. The axes may be:

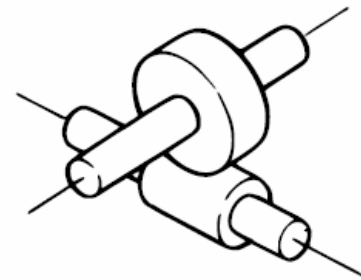
1. **Parallel** shafts where the angle between driving and driven shaft is 0 degree. Examples include spur gears, single and double helical gears.
2. **Intersecting** shafts where there is some angle between driving and driven shaft. Examples include bevel and miter gear.
3. **Non-intersecting and non-parallel** shafts where the shafts are not coplanar. Examples include the hypoid and worm gear.



**Parallel Axis**



**Intersecting Axis**



**Non Parallel  
Non Intersecting Axis**

### According to peripheral velocity:

Gears can be classified as:

1. **Low** velocity type, if their peripheral velocity lies in the range of 1 to 3 m/sec.
2. **Medium** velocity type, if their peripheral velocity lies in the range of 3 to 15 m/sec.
3. **High** velocity type, if their peripheral velocity exceeds 15 m/sec.

**According to type of gears:**

Gears can be classified as external gears, internal gears, and rack and pinion.

1. **External gears** mesh externally - the bigger one is called “gear” and the smaller one is called “pinion”.
2. **Internal gears** mesh internally - the larger one is called “annular” gear and the smaller one is called “pinion”.
3. **Rack and pinion type** – converts rotary to linear motion or vice versa. There is a straight line gear called “rack” on which a small rotary gear called “pinion” moves.

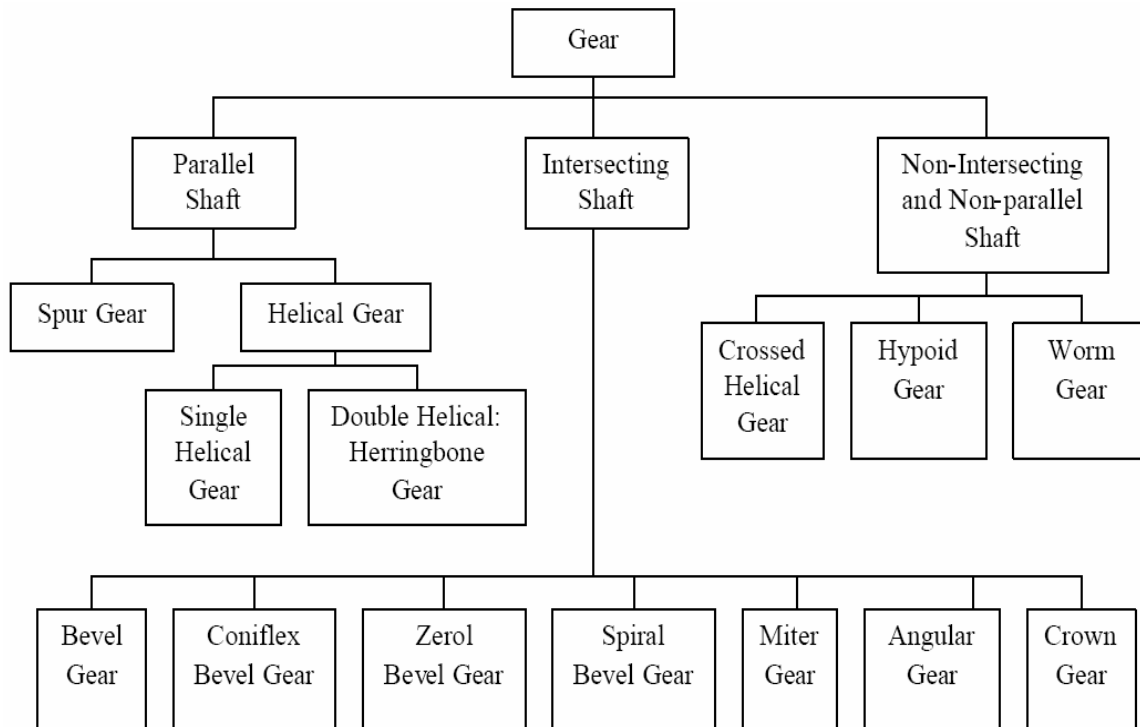
**According to teeth position:**

Gears are classified as straight, inclined and curved.

1. **Straight gear teeth** are those where the teeth axis is parallel to the shaft axis.
2. **Inclined gear teeth** are those where the teeth axis is at some angle.
3. **Curve gear teeth** are curved on the rim’s surface.

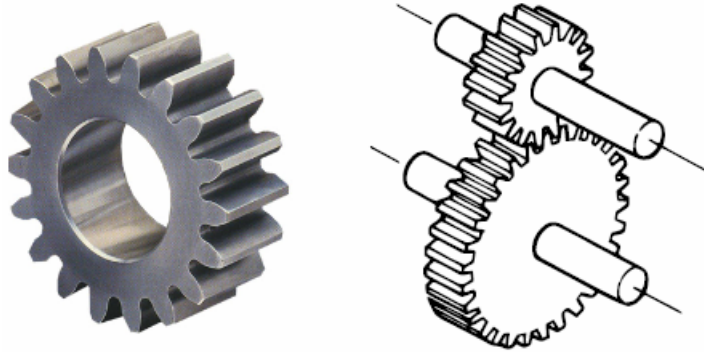
**TYPE OF GEARS**

Here is a brief list of the common forms.



## SPUR GEARS

Spur gears are used to transmit power between two parallel shafts. The teeth on these gears are cut straight and are parallel to the shafts to which they are attached.



Spur Gears

### Characteristics:

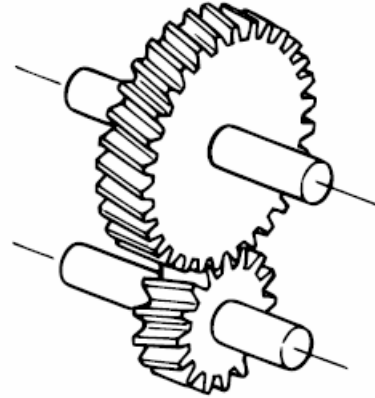
- Simplest and most economical type of gear to manufacture.
- Speed ratios of up to 8 (in extreme cases up to 20) for one step (single reduction) design; up to 45 for two step design; and up to 200 for three-step design.

### Limitations:

- Not suitable when a direction change between the two shafts is required.
- Produce noise because the contact occurs over the full face width of the mating teeth instantaneously.

## HELICAL GEARS

Helical gears resemble spur gears, but the teeth are cut at an angle rather than parallel to the shaft axis like on spur gears. The angle that the helical gear tooth is on is referred to as the helix angle. The angle of helix depends upon the condition of the shaft design and relative position of the shafts. To ensure that the gears run smoothly, the helix angle should be such that one end of the gear tooth remains in contact until the opposite end of the following gear tooth has found a contact. For parallel shafts, the helix angle should not exceed 20 degrees to avoid excessive end thrust.



### Helical Gears

#### Characteristics:

The longer teeth cause helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer than the teeth of spur gear of equivalent pitch diameter.
- Can carry higher loads than can spur gears because of greater surface contact on the teeth.
- Can be used to connect parallel shafts as well as non-parallel, non-intersecting shafts.
- Quieter even at higher speed and are durable.

#### Limitations:

- Gears in mesh produce thrust forces in the axial directions.
- Expensive compared to spur gears.

### BEVEL GEARS

A bevel gear is shaped like a section of a cone and primarily used to transfer power between intersecting shafts at right angles. The teeth of a bevel gear may be straight or spiral. Straight gear is preferred for peripheral speeds up to 1000 feet per minute; above that they tend to be noisy.



**Characteristics:**

- Designed for the efficient transmission of power and motion between intersecting shafts. A good example of bevel gears is seen as the main mechanism for a hand drill. As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation.
- Permit a minor adjustment during assembly and allow for some displacement due to deflection under operating loads without concentrating the load on the end of the tooth.

**MITTER GEARS**

Mitter gears are identical to bevel gears with the exception that both gears always have the same number of teeth.



**Characteristics:**

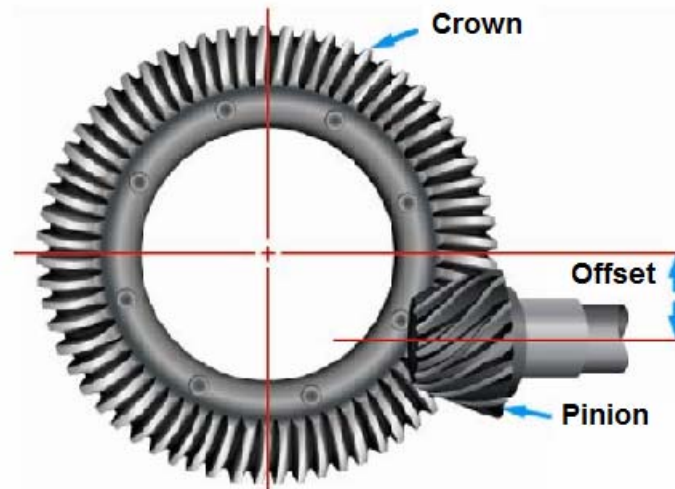
- They provide a steady ratio; other characteristics are similar to bevel gears.
- They are used as important parts of conveyors, elevators and kilns.

**Limitations**

- Gear ration is always 1 to 1 and therefore not used when an application calls for a change of speed.

## HYPOID GEARS

Hypoid gears are a modification of the spiral bevel gear with the axis offset. The distinguishing feature of hypoid gears is that the shafts of the pinion and ring gear may continue past each other, never having their axis intersecting.



Hypoid Gears

The major advantages of the hypoid gear design are that the pinion diameter is increased, and it is stronger than a corresponding bevel gear pinion. The increased diameter size of the pinion permits the use of comparatively high gear ratios and is extremely useful for non-intersecting shaft requirements such as automotive applications where the offset permits lowering of the drive shaft.

## WORM GEARS

Worm gears are used to transmit power between two shafts that are at right angles to each other and are non-intersecting.

Worm gears are special gears that resemble screws, and can be used to drive spur gears or helical gears. Worm gearing is essentially a special form of helical gearing in which the teeth have line contact and the axes of the driving and driven shafts are usually at right angles and do not intersect.





**Worm Gear**

**Characteristics:**

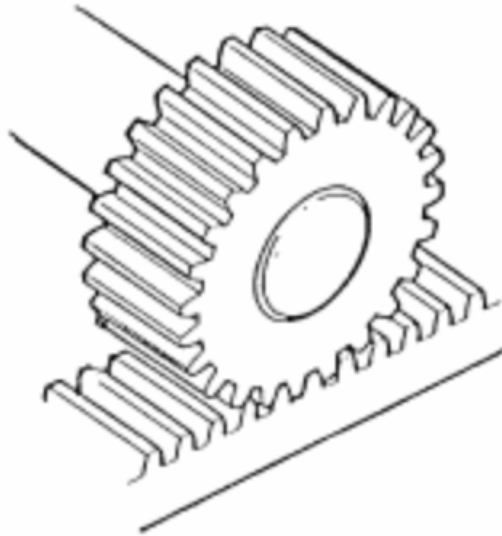
- Meshes are self-locking. Worm gears have an interesting feature that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place. This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning.
- Worm gear is always used as the input gear, i.e. the torque is applied to the input end of the worm shaft by a driven sprocket or electric motor.
- Best suited for applications where a great ratio reduction is required between the driving and driven shafts. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

**Limitations:**

- Yield low efficiency because of high sliding velocities across the teeth, thereby causing high friction losses.
- When used in high torque applications, the friction causes the wear on the gear teeth and erosion of the restraining surface.

**RACKS (STRAIGHT GEARS)**

The rack is a bar with a profile of the gear of infinite diameter, and when used with a meshing pinion, enables the rotary to linear movement or vice versa.



**Characteristics:**

- Racks with machined ends can be joined together to make any desired length.
- The most well-known application of a rack is the rack and pinion steering system used on many cars in the past. The steering wheel of a car rotates the gear that engages the rack. The rack slides right or left, when the gear turns, depending on the way we turn the wheel. Windshield wipers in cars are powered by a rack and pinion mechanism.

**HERRINGBONE (DOUBLE HELICAL) GEARS**

Herringbone, also known as double helical gears, are used for transmitting power between two parallel shafts. Double helical gearing offers low noise and vibration along with zero net axial thrust.



**Herringbone Gears**

**Characteristics:**

- Conduct power and motion between non-intersecting, parallel axis that may or may not have center groove with each group making two opposite helices. Action is equal in force and friction on both gears and all bearings, and free from any axial force.
- Offer reduced pulsation due to which they are highly used for specialized extrusion and polymerization. The most common application is in heavy machinery and power transmission.
- Applications include high capacity reduction drives like that of cement mills and crushers.

**Limitations:**

- Manufacturing difficulty makes them costlier.
- Noise level of double helical gears averaged about 4dB higher than otherwise similar single helical gears. The phenomenon is due to the axial shuttling which occurs as the double helical pinion moves to balance out the net thrust loading.

**INTERNAL GEAR**

Internal gears have their teeth cut parallel to their shafts like spur gears, but they are cut on the inside of the gear blank. The properties and teeth shape are similar as the external gears except that the internal gears have different addendum and dedendum values modified to prevent interference in internal meshes.



**Internal Gear**

**Characteristics:**

- In the meshing of two external gears, rotation goes in the opposite direction. In the meshing of an internal gear with an external gear the rotation goes in the same direction.
- The meshing arrangement enables a greater load carrying capacity with improved safety (since meshing teeth are enclosed) compared to equivalent external gears.
- Shaft axes remain parallel and enable a compact reduction with rotation in the same sense. Internal gears are not widely available as standard.
- When they are used with the pinion, more teeth carry the load that is evenly distributed. The even distribution decreases the pressure intensity and increases the life of the gear.
- Allows compact design since the center distance is less than for external gears. Used in planetary gears to produce large reduction ratios.
- Provides good surface endurance due to a convex profile surface working against a concave surface.

**Applications:**

- Planetary gear drive of high reduction ratios, clutches, etc.

**Limitations:**

- Housing and bearing supports are more complicated because the external gear nests within the internal gear.
- Low ratios are unsuitable and in many cases impossible because of interferences.
- Fabrication is difficult and usually special tooling is required.

**SUMMARY**

Type	Features	Applications	Comments Regarding Precision
Spur	Parallel Shafting. Adapted to high speed applications where noise is not a concern.	Applicable to all types of trains and a wide range of velocity ratios.	Simplest tooth elements offering maximum precision. First choice, recommended for all the gear meshes, except where very high speeds and loads or special features of other types, such as right angle drive, cannot be avoided.
Helical	Parallel Shafting. Very high speeds and loads. Efficiency slightly less than spur mesh.	Most applicable to high speeds and loads; also used whenever spurs are used.	Equivalent quality to spurs, except for complication of helix angle. Recommended for all high-speed and high-load meshes. Axial thrust component must be accommodated.
Crossed Helical	Skewed shafting. Point contact. High sliding Low speeds Light loads	Relatively low velocity ratio; low speeds and light loads only. Any angle skew shafts.	Precision Rating is poor. Point contact limits capacity and precision. Suitable for right angle drives, if light load. A less expensive substitute for bevel gears. Good lubrication essential because of point of contact and high sliding action.

Internal spur	Parallel shafts High speeds High loads	Internal drives requiring high speeds and high loads; offers low sliding and high stress loading; good for high capacity, long life. Used in planetary gears to produce large reduction ratios.	Not recommended for precision meshes because of design, fabrication, and inspection limitations. Should only be used when internal feature is necessary.
Bevel	Intersecting shafts, High speeds, High loads.	Suitable for 1:1 and higher velocity ratios and for right-angle meshes (and other angles)	Good choice for right angle drive, particularly low ratios. However complicated both form and fabrication limits achievement of precision. Should be located at one of the less critical meshes of the train.
Worm mesh	Right-angle skew shafts, High velocity ratio, High speeds and loads, Low efficiency, Most designs nonreversible.	High velocity ratio Angular meshes High loads	Worm can be made to high precision, but worm gear has inherent limitations. To be considered for average precision meshes, but can be of high precision with care. Best choice for combination high velocity ratio and right-angle drive. High sliding requires excellent lubrication.

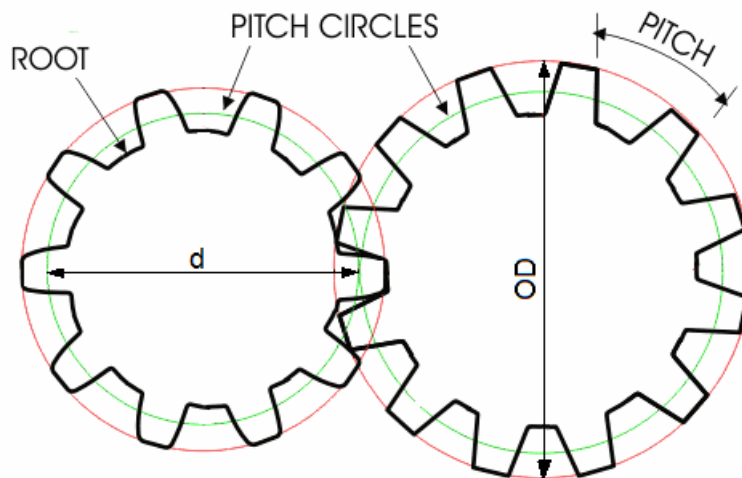
## SECTION -2: GEARS FUNDAMENTALS & TERMINOLOGY

In this section, we will discuss the gear fundamentals, considering spur gears. It is the most common and the simplest form, and hence the most comprehensible. The same principles apply to spiral gears and bevel gears too.

A gear can be defined in terms of its pitch, pressure angle and number of teeth. Let's discuss few terms here:

**Pitch Circle Diameter (d)** - This is the diameter of a circle about which the gear tooth geometry is designed or constructed. The pitch circle is the imaginary circle found at the point where the teeth of two gears mesh. The diameter of the pitch circle is called the pitch diameter.

**Outside Diameter (OD)** - The outside circle is the distance around the outer edge of the gear's teeth. The diameter of the outside circle is called the outside diameter.



**Root** - The root is the bottom part of a gear wheel.

**Pitch** - Pitch is a measure of tooth spacing along the pitch circle. It is the distance between any point on one tooth and same point on the next tooth. It is expressed in the following forms:

**Diametral Pitch ( $P_d$ )** is the number of teeth per inch of the pitch diameter and is also an index of tooth size. It is given as:

$$P_d = \frac{Z}{d}$$

Where:

- $P_d$  = diametral pitch
- $Z$  = number of teeth
- $d$  = pitch circle diameter in inches

A large diametral pitch indicates a small tooth and vice versa. Another way of saying this; larger gears have fewer teeth per inch of diametral pitch.

### Important!

The use of diametral pitch is a handy reference in gear design. An important rule to remember is that a pair of gears can only mesh correctly if and when the diametral pitch ( $P_d$ ) is the same, i.e.:

$$P_d = \frac{Z_{\text{Gear}}}{d_{\text{Gear}}} = \frac{Z_{\text{Pinion}}}{d_{\text{Pinion}}}$$

**Module (m)** is the metric equivalent of diametral pitch, i.e. the pitch diameter (in mm) divided by the number of teeth, but unlike diametral pitch, the higher number, the larger the teeth. Meshing gears must have the same module:

$$m = \frac{1}{P_d} = \frac{d}{Z}$$

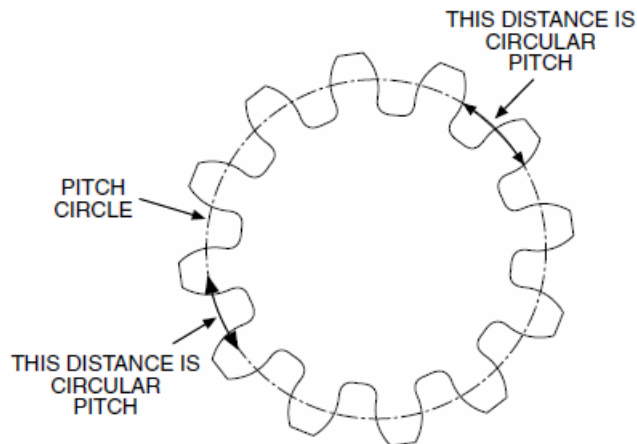
A 1 module gear has 1 tooth for every mm of pitch circle diameter. Thus a 0.3 mod gear having 60 teeth will have a pitch circle diameter of 18 mm ( $0.3 * 60$ ).

**Circular Pitch (P<sub>c</sub>):** is the distance from a point on one tooth to the corresponding point on the adjacent tooth, measured along the pitch circle. Calculated in inches, the circular pitch equals the pitch circle circumference divided by the number of teeth:

$$P_c = \frac{\text{Circumference } (\pi d)}{\text{Number of teeth } (Z)}$$

Because the circular pitch is directly proportional to the module and inversely proportional to the diametral pitch, meshing teeth must have the same circular pitch.





### Relationship between Circular Pitch and Diametral Pitch:

$$\text{From } P_c = \frac{\pi d}{Z} \quad \text{and} \quad P_d = \frac{Z}{d}$$

We have,

$$P_d P_c = \pi$$

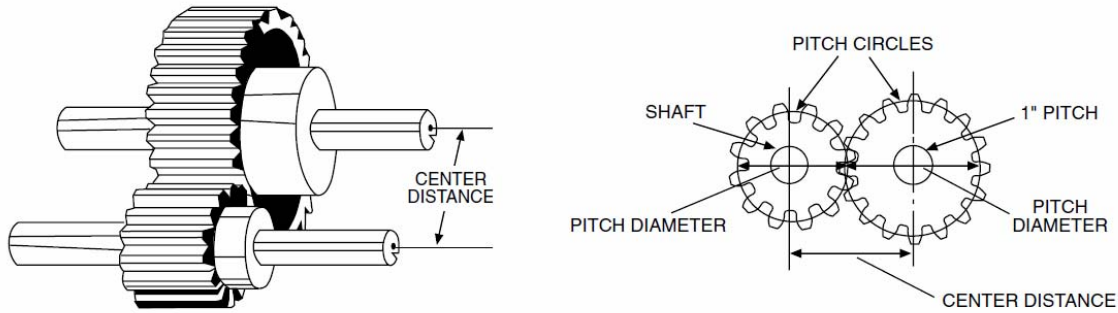
The product of the circular pitch and the diametral pitch is equal to pi ( $\pi$ ).

**Number of Teeth (N):** The number of gear teeth is related to the diametral pitch and the pitch circle diameter by equation  $Z = d \times P_d$ .

**Tooth Size:** Diametral pitch, module and circular pitch are all indications of tooth size; ratios which determine the number of teeth in a gear for a given pitch diameter. In designing a gear set, the number of teeth in each member is of necessity. *As a rule of thumb, teeth should be large and low in number for heavily loaded gears and small and numerous for smooth operation.*

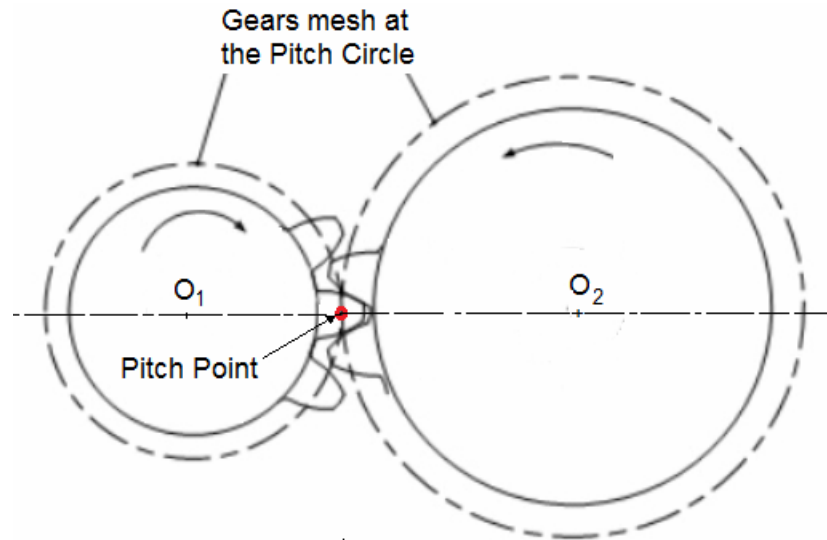
### Center Distance (CD)

Center Distance is the distance between the centers of the shaft of one spur gear to the center of the shaft of the other spur gear. The standard center distance between two spur gears is one-half the sum of their pitch diameters.



**Pitch point:**

Pitch point is the point where gear teeth actually make contact with each other as they rotate. Refer to the figure below for two meshing gears. The pitch point “P” always lies at the line connecting the centers of two gears.



**EXAMPLES**

**Example - 1:**

The center distance of a 4-inch pitch diameter gear running with a 2-inch pitch diameter pinion is 3 inches.  $4" + 2" \div 2 = 3"$  CD

**Example -2:**

A gear has 18 teeth (Z) and a diametral pitch ( $P_d$ ) of 8. What is its pitch diameter (d)?

**Answer:**

$$d = Z/P_d = 18/8 = 2\frac{1}{4}"$$

**Example -3:**

A gear has a pitch diameter (d) of 3.125" (3-1/8") and a diametral pitch ( $P_d$ ) of 8. How many teeth (Z) does it have?

**Answer:**

$$Z = d \times P_d = 3.125 \times 8 = 25 \text{ teeth}$$

**Example -4:**

Calculate the center-to-center spacing for the 2 gears specified below.

Gear 1: 36 tooth, 24  $P_d$  Drive Gear

Gear 2: 60 tooth, 24  $P_d$  Driven Gear

**Answer:**

Calculate the pitch diameter for each of the two gears:

$$\text{Pitch diameter of gear 1: } d_1 = Z / P_d = 36/24 = 1.5''$$

$$\text{Pitch diameter of gear 2: } d_2 = Z / P_d = 60/24 = 2.5''$$

Obtain center to center distance by adding the two diameters and divide by 2.

$$\text{Center to center distance} = (d_1 + d_2) / 2 = (1.5 + 2.5) / 2 = 2''$$

### THE LAW OF GEARING

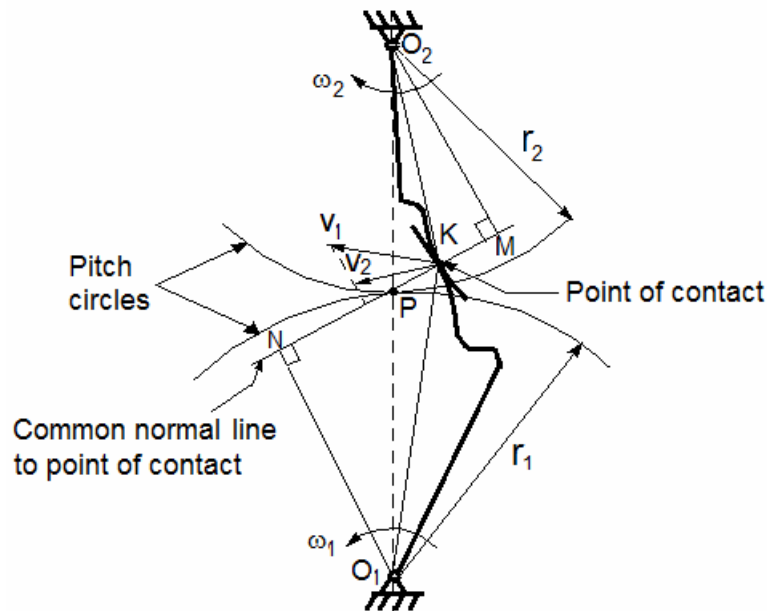
The fundamental law of gearing states that the angular velocity ratio of all gears must remain constant throughout the gear mesh. This condition is satisfied when the common normal at the point of contact between the teeth passes through a fixed point on the line of centers, known as the pitch point.

#### Law Governing Shape of the Teeth

The figure below shows two mating gear teeth in which:

Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point K.

- NM is the common normal of the two profiles.
- N is the foot of the perpendicular from  $O_1$  to NM
- M is the foot of the perpendicular from  $O_2$  to NM.



Although the two profiles have different velocities  $v_1$  and  $v_2$  at point  $K$ , their velocities along  $NM$  are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other.

$$O_1N \cdot \omega_1 = O_2M \cdot \omega_2$$

$$\omega_1/\omega_2 = O_2M/O_1N$$

We notice that the intersection of the tangency  $NM$  and the line of center  $O_1O_2$  is point  $P$ , and  $O_1N \cdot P = O_2M \cdot P$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or velocity ratio, of a pair of mating teeth is:

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{r_2}{r_1} = \frac{d_2}{d_1}$$

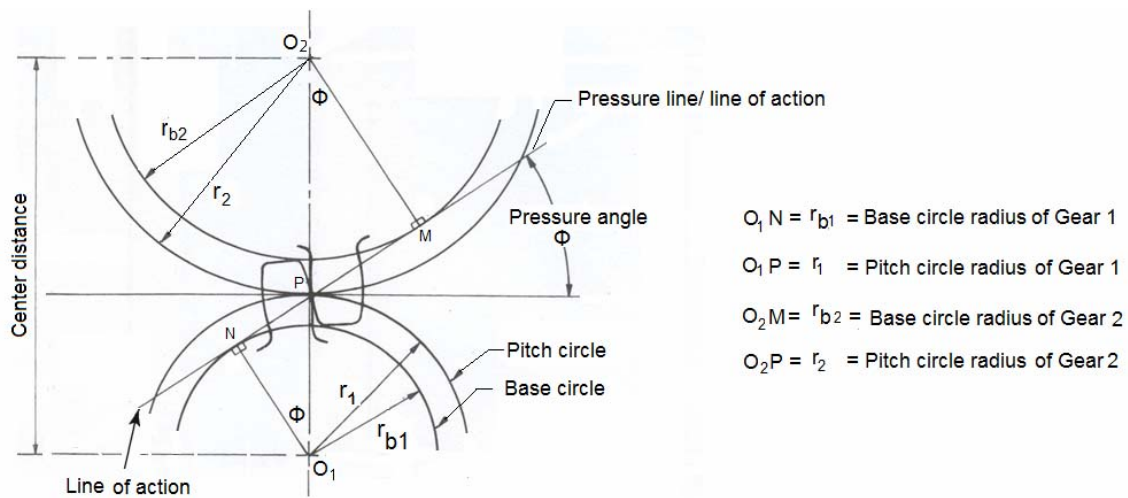
Where,

	<b>Pinion</b>	<b>Gear</b>
Radius	$r_1$	$r_2$
Diameter (inches)	$d_1$	$d_2$
Speed (rad/s)	$\omega_1$	$\omega_2$

Point P is very important to the velocity ratio, and it is called the pitch point. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

When the tooth profiles of gears are shaped so as to produce a constant angular-velocity ratio during meshing, the surfaces are said to be conjugate. **Pressure angle** defines the shape of the gear tooth and is an important criterion in gear manufacturing. Higher pressure angle results in wider base, stronger teeth and lower tendency to experience tooth tip interference, but are susceptible to noise and higher bearing loads. Low pressure angles are quieter and smoother, have lower bearing loads and lower frictional forces, but are susceptible to undercutting at low number of teeth.

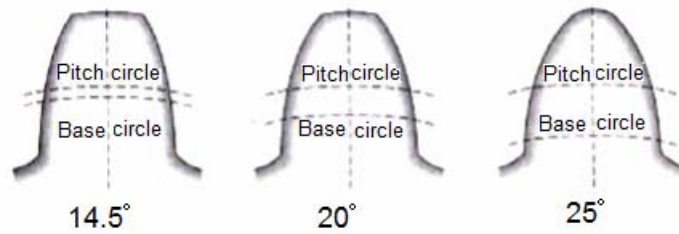
Refer to the figure below:



Draw radial lines from the center of each gear  $O_1$  &  $O_2$  that are perpendicular to the line of action MN. The normal to the line of action up to the center of gears,  $O_1N$  and  $O_2M$ , can be used to form a circle of radius  $r_{b1}$  &  $r_{b2}$  referred to as the base circles of gear 1 and gear 2, respectively. The base circle is inscribed within the pitch circle having radius  $r_1$  and  $r_2$ . As is evident from the geometry of the figure, the angle between the line of centers ( $O_1O_2$ ) and the line segment  $O_1N$  and  $O_2M$  is the pressure angle  $\Phi$ . It is also equal to the angle between the line of action MN and the line perpendicular to the line of centers ( $O_1O_2$ ) through the pitch point P.

The standard pressure angles are  $14\frac{1}{2}^\circ$ ,  $20^\circ$  and  $25^\circ$ . The preferred angle in use today is  $20^\circ$ ; a good compromise for power and smoothness. The increase of the pressure

angle from  $14\frac{1}{2}^\circ$  to  $25^\circ$  results in a stronger tooth, because the tooth acting as a beam is wider at the base.



**Gear Tooth Profile for different Pressure Angles**

### **Important!**

*It is important to note that the gears must have the same pressure angles to mesh.  $14\frac{1}{2}^\circ$  PA tooth forms will not mesh with  $20^\circ$  pressure angles gears and vice versa.*

### **Contact Ratio:**

In the above description, we have considered one gear tooth in contact for simplicity. In practice, more than one tooth is actually in contact during engagement and therefore the load is partially shared with another pair of teeth. This property is called the **contact ratio**. A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact, and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact.

The higher the contact ratio the more the load is shared between teeth. It is a good practice to maintain a contact ratio of 1.3 to 1.8. Under no circumstances should the ratio drop below 1.1.

## **GEAR PROFILES**

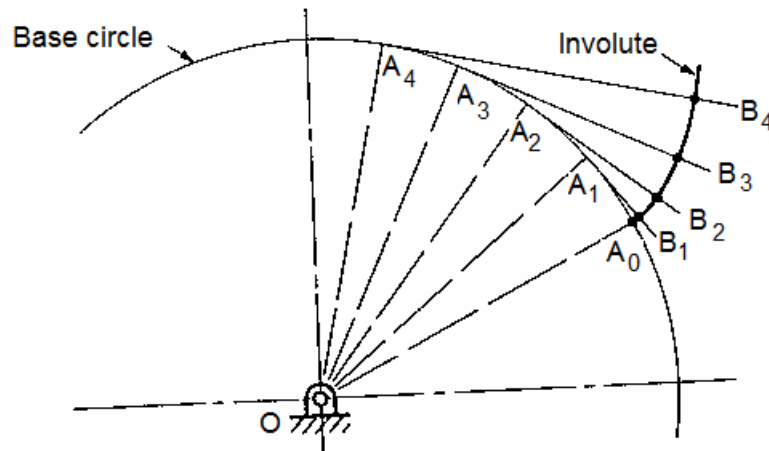
Gear profiles should satisfy the law of gearing. The profiles best suited for this law are:

1. Involute
2. Cycloidal

### **Involute Tooth Profile**

Most modern gears use a special tooth profile called an involute. This profile has the very important property of maintaining a constant speed ratio between the two gears.

The involute profile is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the base circle.



We use the word *involute* because the contour of gear teeth curves inward. On an involute gear tooth, the contact point starts closer to one gear, and as the gear spins, the contact point moves away from that gear and towards the other. Involute gears have the invaluable ability of providing conjugate action when the gears' centre distance is varied either deliberately or involuntarily due to manufacturing and/or mounting errors.

### Cycloidal Tooth Profile

Cycloidal gears have a tooth shape based on the epicycloid and hypocycloid curves, which are the curves generated by a circle rolling around the outside and inside of another circle, respectively. They are not straight and their shape depends on the radius of the generating circle. Cycloidal gears are used in pairs and are set at an angle of 180 degrees to balance the load. The input and output remains in constant mesh. Cycloidal tooth forms are used primarily in clocks for a number of reasons:

- Less sliding friction
- Less wear
- Easier to achieve higher gear ratios without tooth interference

### Comparison between Involute and Cycloidal Gears

In actual practice, the involute tooth profile is the most commonly used because of following advantages:

1. The most important advantage of the involute gears is that the variations in center distance do not affect the angular velocity ratio. This is not true for cycloidal gears which require exact center distance to be maintained.
2. In involute gears, the pressure angle remains constant throughout the engagement of teeth which results in smooth running. The involute system has a standard pressure angle which is either  $20^\circ$  or  $14\frac{1}{2}^\circ$ , whereas on a cycloidal system, the pressure angle varies from zero at pitch line to a maximum at the tips of the teeth.
3. The face and flank of involute teeth are generated by a single curve, whereas in cycloidal gears, double curves (i.e. epicycloid and hypo-cycloid) are required for the face and flank, respectively.
4. Cycloidal teeth have wider flanks; therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred especially for cast gears used in paper mill machinery and sugar mills.
5. Cycloidal gears do not have interference.

Though there are advantages of cycloidal gears, they are outweighed by the greater simplicity and flexibility of the involute gears. It is easy to manufacture since it can be generated from a simple cutter.

The only disadvantage of the involute teeth is that the interference occurs with pinions having smaller number of teeth. This may be avoided by altering the heights of addendum and dedendum of the mating teeth, or the angle of obliquity of the teeth.

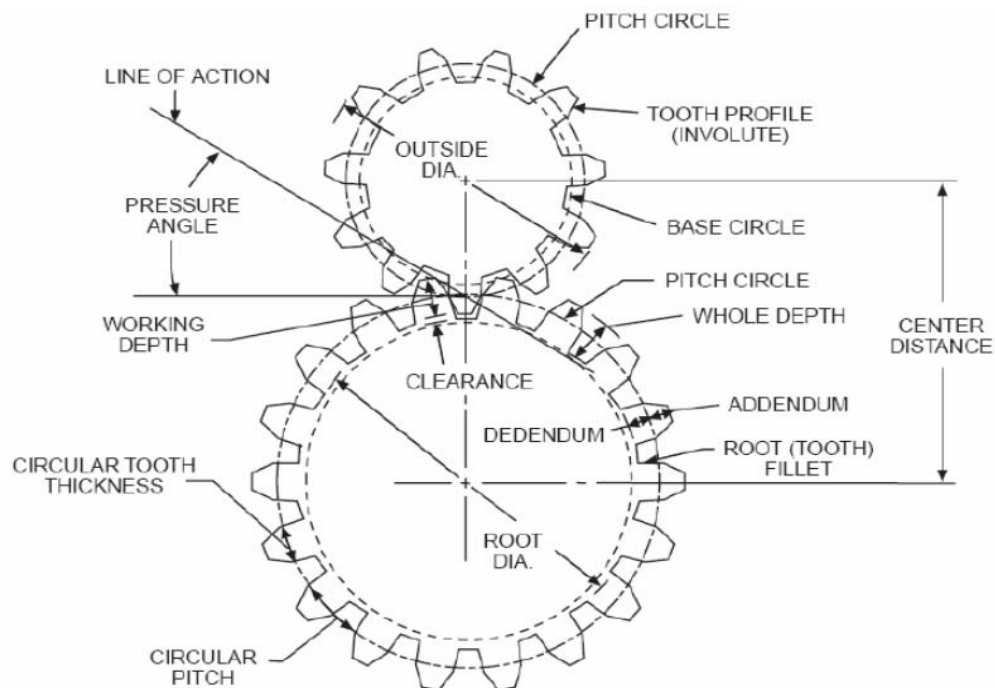
### GEAR TOOTH NOMENCLATURE

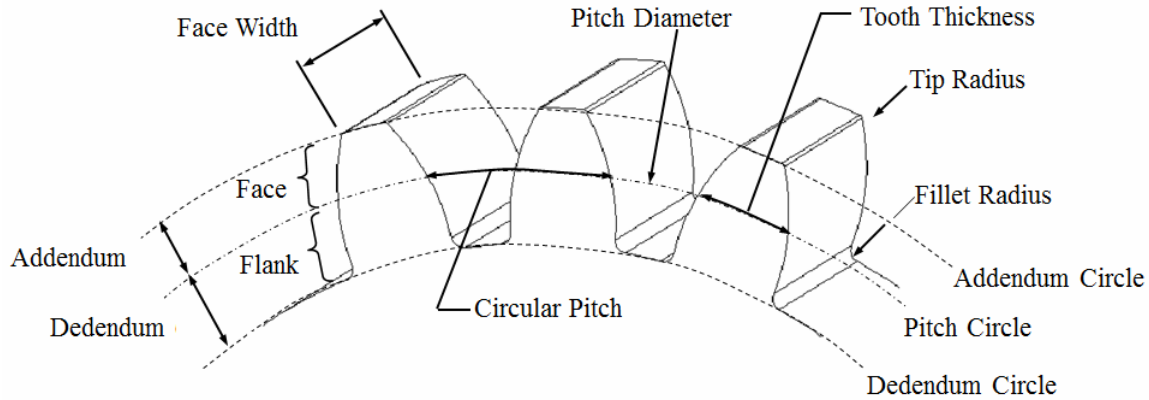
The following terms are used when describing the dimensions of a gear tooth:

1. **Addendum:** the distance from the top of a tooth to the pitch circle. Its value is equal to one module.



2. **Dedendum:** the distance from the pitch circle to the bottom of the tooth space (root circle). It equals the addendum + the working clearance. Dedendum is bigger than addendum and is equal to  $\text{Addendum} + \text{Clearance} = m + 0.157m = 1.157m$
3. **Whole depth:** The total depth of the space between adjacent teeth and is equal to addendum plus dedendum. Also equal to working depth plus clearance.
4. **Working depth:** Working depth is the depth of engagement of two gears; that is, the sum of their addendums.
5. **Working Clearance:** This is a radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Its standard value is  $0.157m$ .
6. **Outside diameter:** The outside diameter of the gear.
7. **Base Circle diameter:** The diameter on which the involute teeth profile is based.
8. **Addendum circle:** A circle bounding the ends of the teeth in a right section of the gear.
9. **Dedendum circle:** The circle bounding the spaces between the teeth in a right section of the gear.





10. **Tooth space:** It is the width of space between two teeth measured on the pitch circle.

11. **Face of tooth:** It is that part of the tooth surface which is above the pitch surface.

12. **Flank of the tooth:** It is that part of the tooth surface which is lying below the pitch surface.

13. **Point of contact:** Any point at which two tooth profiles touch each other.

14. **Path of action:** The locus of successive contact points between a pair of gear teeth, during the phase of engagement.

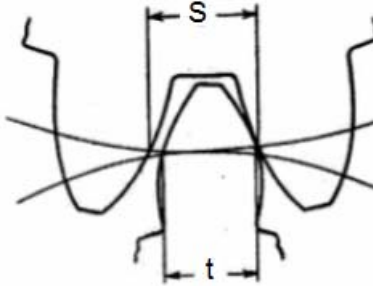
15. **Line of action:** The line of action is the path of action for involute gears. It is the straight line passing through the pitch point and tangent to both base circles.

16. **Tooth Thickness, Space Width and Backlash**

- **Tooth thickness, (t)** is the width of the tooth (arc distance) measured on the pitch circle.
- **Space width, (S)** or tooth space is the arc distance between two adjacent teeth measured on the pitch circle.
- **Backlash, (B)** is the difference between the space width and the tooth thickness.

$$B = S - t$$

Standard gears are designed with a specified amount of backlash to prevent noise and excessive friction and heating of the gear teeth.



### SPUR GEAR FORMULAS AND CALCULATIONS

Below is a table of formulas used in calculating spur gear information based on standard gearing practices. The spur gear formulas here are based on the "Diametral Pitch system ( $P_d$ ).

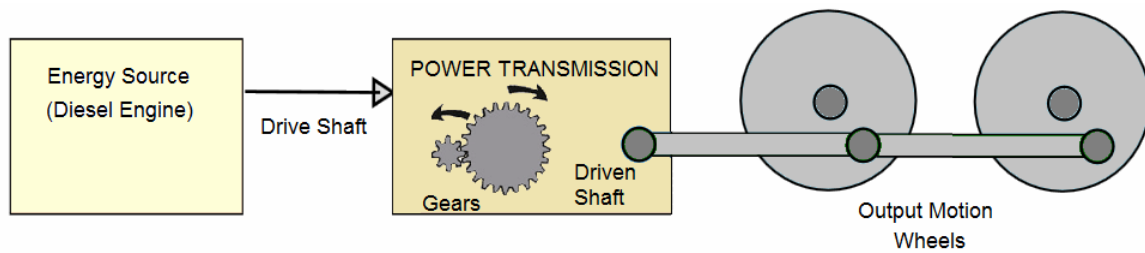
To Get	Having	Rule	Formula
Diametral Pitch	<ul style="list-style-type: none"> <li>The circular pitch</li> </ul>	Divide Pi (3.1416) by the circular pitch	$P_d = 3.1416 / CP$
Diametral Pitch	<ul style="list-style-type: none"> <li>Pitch diameter</li> <li>Number of teeth</li> </ul>	Divide the number of teeth by the pitch diameter	$P_d = Z / d$
Diametral Pitch	<ul style="list-style-type: none"> <li>Outside diameter</li> <li>Number of teeth</li> </ul>	Divide number of teeth + 2 by the outside diameter	$P_d = (Z+2) / OD$
Diametral Pitch	<ul style="list-style-type: none"> <li>Base pitch</li> <li>Pressure angle</li> </ul>	Divide the base pitch by the cosine of the pressure angle then divide by 3.1416	$P_d = (BP / \text{Cos}\Phi) / 3.1416$
Diametral Pitch	<ul style="list-style-type: none"> <li>Module</li> </ul>	Divide 25.4 by Module	$P_d = 25.40 / M$
Pitch Diameter	<ul style="list-style-type: none"> <li>Number of teeth</li> <li>Diametral pitch</li> </ul>	Divide the number of teeth by the diametral pitch	$d = Z / P_d$
Pitch Diameter	<ul style="list-style-type: none"> <li>Number of teeth</li> <li>Outside diameter</li> </ul>	Divide the product of the outside diameter + number of teeth by the number of teeth + 2	$d = (OD \times Z) / (Z+2)$
Pitch Diameter	<ul style="list-style-type: none"> <li>Outside diameter</li> <li>Diametral pitch</li> </ul>	Subtract 2 divided by the diametral pitch from the outside diameter	$d = OD - (2/ P_d)$

Pitch Diameter	<ul style="list-style-type: none"> <li>• Addendum</li> <li>• Number of teeth</li> </ul>	Multiply addendum by the number of teeth	$d = a \times Z$
Pitch Diameter	<ul style="list-style-type: none"> <li>• Base diameter</li> <li>• Pressure angle</li> </ul>	Divide the base diameter by the cosine of the pressure angle	$d = BD / (\text{Cos}\Phi)$
Outside Diameter	<ul style="list-style-type: none"> <li>• Number of teeth</li> <li>• Diametral pitch</li> </ul>	Divide number of teeth + 2 by the diametral pitch	$OD = (Z+2) / P_d$
Outside Diameter	<ul style="list-style-type: none"> <li>• Pitch diameter</li> <li>• Diametral pitch</li> </ul>	Two divided by the diametral pitch plus pitch diameter	$OD = (2 / P_d) + d$
Outside Diameter	<ul style="list-style-type: none"> <li>• Pitch diameter</li> <li>• Number of teeth</li> </ul>	Number of teeth + 2, divided by the quotient of number of teeth divided by the pitch diameter	$OD = (Z+2) / (Z / d)$
Outside Diameter	<ul style="list-style-type: none"> <li>• Number of teeth</li> <li>• Addendum</li> </ul>	Multiply the number of teeth + 2 by the addendum	$OD = (Z + 2) \times A$
Number Of Teeth	<ul style="list-style-type: none"> <li>• Pitch diameter</li> <li>• Diametral pitch</li> </ul>	Multiply the pitch diameter by the diametral pitch	$Z = d \times P_d$
Number Of Teeth	<ul style="list-style-type: none"> <li>• Outside diameter</li> <li>• Diametral pitch</li> </ul>	Multiply the outside diameter by the diametral pitch and subtract 2	$Z = (OD \times P_d) - 2$
Thickness Of Tooth	<ul style="list-style-type: none"> <li>• Diametral pitch</li> </ul>	Divide 1.5708 by the diametral pitch	$t = 1.5708 / P_d$
Addendum	<ul style="list-style-type: none"> <li>• Diametral pitch</li> </ul>	Divide 1 by the diametral pitch	$a = 1 / P_d$
Dedendum	<ul style="list-style-type: none"> <li>• Diametral pitch</li> </ul>	Divide 1.157 (or 1.25) by the diametral pitch	$b = 1.157 / P_d$
Working Depth	<ul style="list-style-type: none"> <li>• Diametral pitch</li> </ul>	Divide 2 by the diametral pitch	$hk = 2 / P_d$
Whole Depth	<ul style="list-style-type: none"> <li>• Diametral pitch</li> </ul>	Divide 2.157 (or 2.25) by the diametral pitch	$ht = 2.157 / P_d$

Clearance	<ul style="list-style-type: none"> <li>Diametral pitch</li> </ul>	Divide .157 (or .250) by the diametral pitch	$c = .157 / P_d$
Clearance	<ul style="list-style-type: none"> <li>Diametral pitch</li> </ul>	Divide thickness of tooth at the pitchline by 10	$c = t / 10$
Operating Diametral Pitch	<ul style="list-style-type: none"> <li>C.D. between both gears</li> <li>Number of teeth in each</li> </ul>	Add the number of teeth in both gears and divide by 2 then divide by the center distance	$P_{do} = ((Z1 + Z2)/2) / C$
Center Distance	<ul style="list-style-type: none"> <li>Normal diametral pitch</li> <li>Number of teeth in both gears</li> </ul>	Add the number of teeth in both gears and divide by 2 then divide by the normal diametral pitch	$C = ((Z1 + Z2)/2) / P_{nd}$
Operating Center Distance	<ul style="list-style-type: none"> <li>Operating diametral pitch</li> <li>Number of teeth in both gears</li> </ul>	Add the number of teeth in both gears and divide by 2 then divide by the operating diametral pitch	$Co = ((Z1 + Z2)/2) / P_{od}$
Base Diameter	<ul style="list-style-type: none"> <li>Pitch diameter</li> <li>Pressure angle</li> </ul>	Multiply the pitch diameter by the cosine of the pressure angle	$BD = P_d \times (\cos\Phi)$
Pressure Angle	<ul style="list-style-type: none"> <li>Base diameter</li> <li>Pitch diameter</li> </ul>	Divide the base diameter by the pitch diameter	$\cos\Phi = BD / d$
Pressure Angle	<ul style="list-style-type: none"> <li>Base pitch</li> <li>Diametral pitch</li> </ul>	Divide $P_i$ by the diametral pitch, then divide by the base pitch	$\cos\Phi = (3.1416 / P_d) / P_b$
Base Pitch	<ul style="list-style-type: none"> <li>Diametral pitch</li> <li>Pressure angle</li> </ul>	Divide the diametral pitch by $P_i$ , then multiply by the cosine of the pressure angle	$P_b = (P_d / 3.1416) \times (\cos\Phi)$

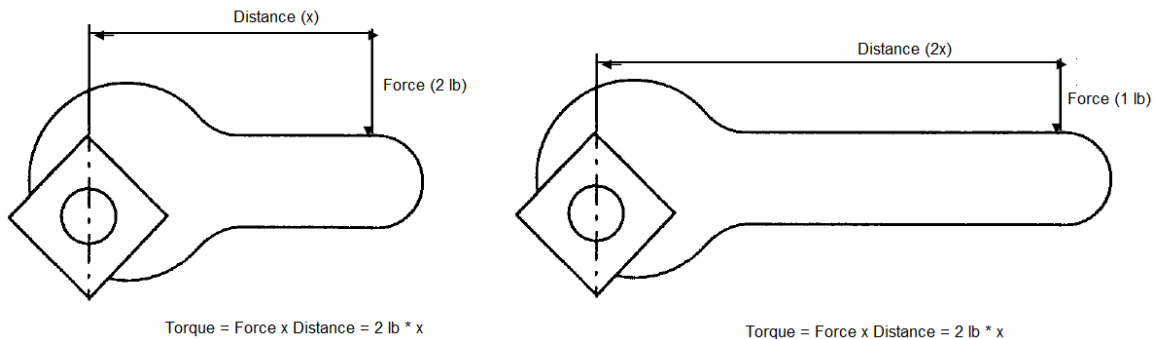
**SECTION -3 POWER TRANSMISSION FUNDAMENTALS**

Power transmission is the transfer of energy from its place of generation to a location where it is applied to performing useful work. Power transmission is normally accomplished by belts, ropes, chains, gears, couplings and friction clutches. Out of these, the gears are capable of transmitting force or motion without any slip and therefore are the most durable and rugged of all mechanical devices. In the schematic below, a gear transmits rotation force from prime mover (diesel engine) to another driven shaft (locomotive wheels).



**Schematic of Diesel Locomotive**

The most important feature of gears is that it produces a mechanical advantage, which is a measure of the force amplification. Since we do not get something for nothing, you can either achieve high velocity output or high force/torque output but not both. The model for this is the law of the lever where a smaller force acting through a greater distance produces the same output as the larger force on a smaller distance.



**Energy and Power Equations**

Power, torque and speed are the defining mechanical variables associated with the functional performance of rotating machinery. Let's do some analysis for gears:

- P = Power
- E = Energy
- W = Work
- F = Force
- T = torque
- d = distance of translational motion
- $\theta$  = angle of rotational motion (in radians)
- v = velocity of translational motion
- $\omega$  = angular speed (in radians per second)
- $\Delta$  = change
- Pd = Pitch diameter
- Z = number of teeth on a gear
- r = Pitch circle radius
- N = number of revolutions

Power is defined as energy per unit of time or the rate at which work is performed and thus:

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}$$

When force (F) moves a body a measured distance ( $\Delta d$ ), the work done (W) is given by:

W (work) = Force x Distance

$$W = F \times \Delta d$$

This equation is true for linear motion but the corresponding definition of work for rotational power transmission is given by the Torque (T) and the angular displacement ( $\Delta\theta$ ). Therefore, work done for rotary motion is:

$$W = T \Delta\theta$$

Rotation is perceived as a change in the angular position of a reference point on the body over some time interval,  $\Delta t$ . The power transfer in a rotary device is therefore given by:

$$P = \frac{W}{\Delta t} = \frac{T \Delta\theta}{\Delta t} \quad \text{Eq. A}$$

The rotary motion is characterized by its angular velocity ( $\omega$ ) and is defined as:

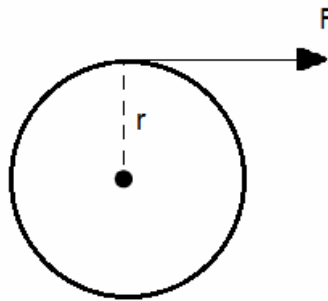
$$\omega = \frac{\Delta\theta}{\Delta t}$$

Substituting the rotary definition of work into Eq. A:

$$P = T \omega \quad \text{Eq. 1}$$

Let's break Torque (T) and angular velocity ( $\omega$ ) in friendly terms.

Torque is a measure of the tendency of a force to rotate an object about some axis. In order to produce torque, the force must act at some distance from the axis or pivot point. In the following diagram, the circle represents a wheel of radius  $r$ ; the dot in the center represents the axle or shaft; and the force (F) is applied tangentially at the periphery.



The amount of torque about the gear axle is:

Torque = Force x Radius

$$T = F \times r \quad \text{Eq. 2}$$

Substituting T (Eq. 2) to Eq. 1:

$$P = F * r * \omega \quad \text{Eq. 3}$$



Angular velocity ( $\omega$ ) is often referred to as rotational speed and measured in numbers of complete revolutions per minute (rpm) or per second (rps). It is usually expressed as:

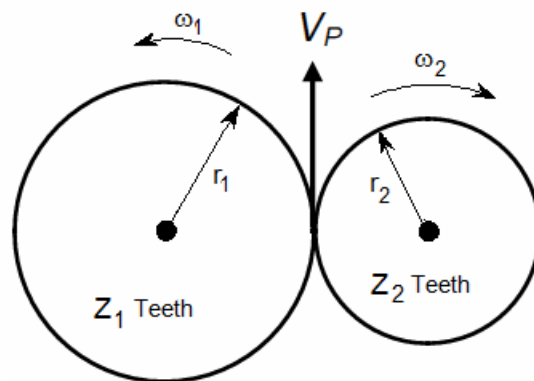
$$\omega = \frac{2\pi N}{60} \quad \left\{ N = \frac{\text{Number of rotations}}{\text{minute}}, \text{ (RPM)} \right\} \quad \text{Eq. 4}$$

Substituting  $\omega$  (Eq. 4) to Eq. 3:

$$P = F * r * \frac{2\pi N}{60} \quad \text{Eq. 5}$$

### ANALYSIS FOR A GEAR PAIR

Consider two gears in a mesh. Gear 1 (driver) is turning counterclockwise at angular velocity  $\omega_1$  and has  $Z_1$  teeth. Gear 2 (the driven gear) is turning clockwise with angular velocity  $\omega_2$  and has  $Z_2$  teeth. The drive between the two gears is represented by plain cylinders having diameters equal to their pitch circles.



We have learnt in Section-2 that a pair of gears can only mesh correctly if and when the diametral pitch ( $P_d$ ) is the same, accordingly:

$$P_d = \frac{Z_1}{d_1} = \frac{Z_2}{d_2} \quad \text{Eq. 6}$$

The driving gear pushes the driven gear, exerting a force component perpendicular to the gear radius; and because the gear is rotating, power is transferred.

$$P_{in} = T_1 \omega_1$$

$$P_{\text{out}} = T_2 \omega_2$$

Assuming no frictional losses, the input and output power can be set equal to each other as:

$$P_{\text{in}} = P_{\text{out}}$$

$$\text{or } \omega_1 T_1 = \omega_2 T_2 \quad \text{Eq. 7}$$

$$\text{or } \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\text{Since, } \omega = \frac{2\pi N}{60} \quad \left\{ N = \frac{\text{Number of rotations}}{\text{minute}}, \text{ (RPM)} \right\}$$

$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{Eq. 8}$$

*It follows that torque and speed are inversely proportional. If a high torque is desired, then the speed must be sacrificed. When speed increases, the torque decreases proportionally.*

We now consider the relative velocity of the two gears. The point of contact of the two pitch surfaces shall have velocity along the common tangent. The velocity of a point on a rotating object is given by  $r \omega$ . Because there is no slip, definite motion of gear 1 can be transmitted to gear 2, therefore:

$$v = \omega_1 r_1 = \omega_2 r_2$$

Where  $r_1$  and  $r_2$  are pitch circle radii of gears 1 and 2, respectively.

$$\omega_1 r_1 = \omega_2 r_2 \quad \text{Eq. 9}$$

$$\text{or } \frac{2\pi N_1}{60} r_1 = \frac{2\pi N_2}{60} r_2$$

$$\text{or } N_1 r_1 = N_2 r_2$$

$$\text{or } \frac{N_1}{N_2} = \frac{r_2}{r_1}$$

Putting it in terms of diameter,  $r = d/2$ , it implies:

$$N_1 \frac{d_1}{2} = N_2 \frac{d_2}{2}$$

or  $N_1 d_1 = N_2 d_2$  Eq. 10

or  $\frac{N_1}{N_2} = \frac{d_2}{d_1}$

*It follows that speed and diameter are inversely proportional. If a high speed is desired, then the diameter of driven gear must be lower than the driving gear.*

Since, pitch circle radius of a gear is proportional to its number of teeth (Z):

$$N_1 \frac{Z_1}{P_d} = N_2 \frac{Z_2}{P_d}$$

or  $N_1 Z_1 = N_2 Z_2$

or  $\frac{N_1}{N_2} = \frac{Z_2}{Z_1}$  Eq. 11

*It follows that the velocity ratio of a pair of gears is the inverse ratio of their number of teeth, i.e. the gear with the greater number of teeth will always revolve slower than the gear with the smaller number of teeth.*

We can now combine the torque equation (Eq. 8), dia. Equation (Eq. 10) and the velocity equation (Eq. 11) to get the relationship with the gear teeth ratio.

$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{T_2}{T_1} = \frac{d_2}{d_1} = \frac{Z_2}{Z_1}$$

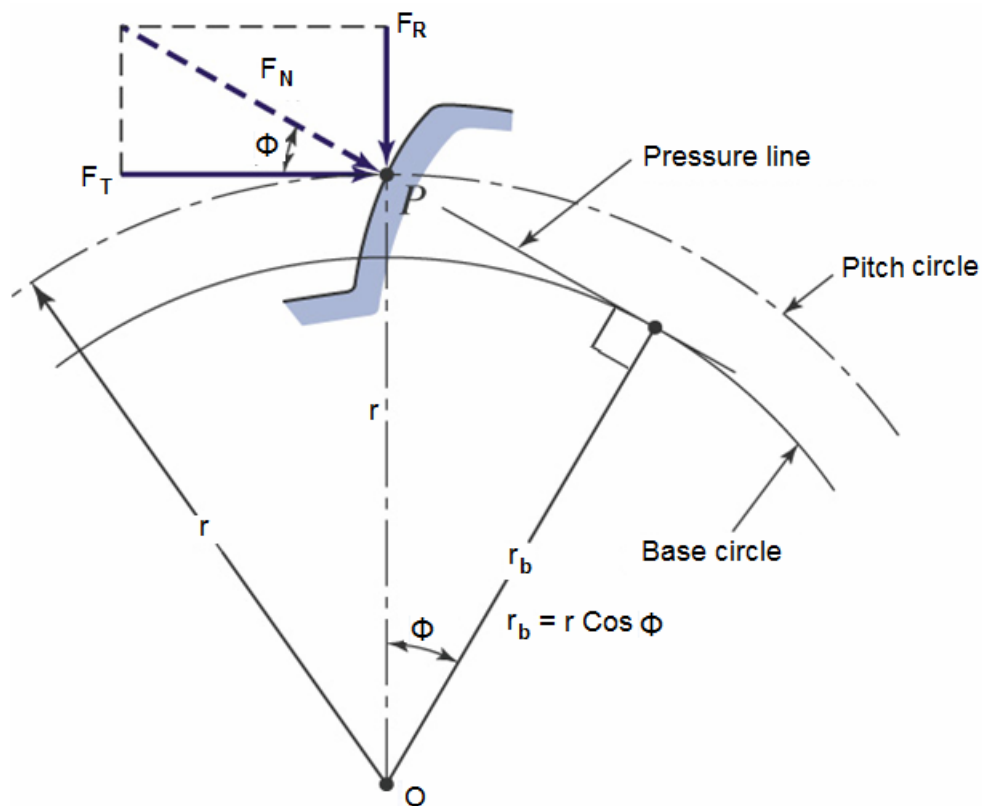
*Gear ratio is defined as the ratio of diameters (teeth) of output to the input gear. When the input gear is smaller than the output gear, the output torque is higher than the input torque and the output speed is lower than the input speed or in other words “a higher gear ratio equates to high torque and lower speed”.*

Let's compare this analogy to a car engine gearbox. In top gears, one turn of the engine crankshaft results in one turn of the drive wheels. Lower gears require more turns of the engine to provide single turn of the drive wheels, producing more torque at the drive wheel. For example, if the driving gear has 10 teeth and the driven gear has 20 teeth, the gear ratio is 2 to 1. Every revolution of the driving gear will cause the driven gear to

revolve through only half a turn. Thus, if the engine is operating at 2,000 rpm, the speed of the driven gear will be only 1,000 rpm; the speed ratio is then 2 to 1. This arrangement doubles the torque on the shaft of the driven unit. The speed of the driven unit, however, is only half that of the engine. On the other hand, if the driving gear has 20 teeth and the driven gear has 10 teeth, the speed ratio is 1 to 2, and the speed of the driven gear is doubled. The rule applies equally well when an odd number of teeth is involved. If the ratio of the teeth is 37 to 15, the speed ratio is slightly less than 2.47 to 1. In other words, the driving gear will turn through almost two and a half revolutions while the driven gear makes one revolution.

### POWER FLOW THROUGH GEAR PAIR

With a pair of gears, power is transmitted by the force developed between contacting teeth. According to fundamental law of gear this resultant force always acts along the pressure line (or the line of action). To investigate how the forces are typically transmitted between a pair of gears, refer to the figure below.



**Power Flow through a Gear Pair**

**(View showing tangential and radial forces on the gear)**

The force transmitted along the line of action results in a torque generated at the base circle. The torque can be calculated by:

$$T = F_N r_b \quad \text{Eq. 12}$$

From the figure above, the relationship between the base circle and pitch circle radius can be stated as:

$$r_b = r \cos\Phi$$

Or

$$T = F_N r \cos\Phi \quad \text{Eq. 13}$$

Where,

- $F_N$  is the force action on the line of action
- $\Phi$  is the pressure angle
- $r_b$  is the radius of base circle
- $r$  is the pitch circle radius of the gear

This resultant force  $F_N$  can be resolved into two components: tangential component  $F_T$  and radial components  $F_R$  at the pitch point.

- $F_T$  - the tangential force component acting at the radius of the pitch circle. It determines the magnitude of the torque and consequently the power transmitted. The Tangential component is expressed as:  **$F_T = F_N \cos\Phi$**
- $F_R$  - the radial or normal force directed towards the center of the gear.  $F_R$  serves to separate the shafts connected to the gears and for this reason  $F_R$  is sometimes referred to as the separating force. The radial component is expressed as:  **$F_R = F_N \sin\Phi$  or  $F_T \tan\Phi$**

Torque exerted on the gear shaft in terms of the pitch circle radius can be found by substituting  $F_T = F_N \cos\Phi$  in Eq. 13. The resultant expression is:

$$T = F_T \times r$$

Or

$$T = F_T \times \frac{d}{2} \quad \text{Eq. 14}$$

The maximum force ( $F_T$ ) is exerted along the common normal through the pitch point which is line perpendicular to the line of centers. After determining  $F_T$ , the magnitude of the other force components and/or their directions can be readily determined.

Recall the following relationship existing between speed, torque and power:

$$P = T \omega$$

$$T = \frac{P}{\omega}$$

$$\text{Since, } \omega = \frac{2 \pi N}{60}$$

The torque transmitted by the gear is given by:

$$T = P \times \frac{60}{2 \pi N}$$

Where,

- T = Torque transmitted gears (N- m)
- P = Power transmitted by gears (kW)
- N = Speed of rotation (RPM)

Alternatively in US units:

$$T = P \times \frac{63000}{N}$$

Eq. 15

Where,

- P = Power, HP
- T = Torque in-lbs
- N = RPM

Substituting Eq. 14 into Eq. 15

$$F_T \times \frac{d}{2} = P \times \frac{63000}{N}$$

Or

$$F_T = P \times \frac{63000}{N} \times \frac{2}{d}$$

Or

$$F_T = P \times \frac{126000}{N d}$$

Power can also be expressed in terms of the pitch line velocity  $v$ .

$$P = F_T v$$

- $P$  = Power in watts
- $F_T$  = force, N
- $v$  = velocity, m/s

Or in terms of customized units

$$P = F_T v \left[ \frac{1m}{60s} \right] \left[ \frac{1 \text{ H.P}}{550 \text{ ft-lbs/s}} \right] = \frac{F_T v}{33,000}$$

- $v$  = velocity in ft/min
- $F_T$  = force in lbs
- $P$  = Power in HP

The above expression can be rearranged to solve for  $F_T$ ;

$$F_T = \frac{33,000 P}{v}$$

The velocity  $v$ :

$$v = .262 \times d \times N$$

Where,

- $d$  = Pitch diameter
- $N$  = Revolutions per minute, RPM

**Important!**

If the forces transmitted between the teeth of meshing gears are transmitted along the line of action at every point of contact, then regardless of the angular position of the gears, the forces transmitted between the gears maintain a fixed orientation in space. Maintaining a fixed orientation in space for the forces to be transmitted between gears

enables the power transmitted between the gears to be independent of the angular position of the gears. This is a very desirable characteristic for gears.

### Example

20-tooth, 8 pitch, 1-inch-wide, 20° pinion transmits 5 HP at 1725 rpm to a 60-tooth gear. Determine driving force, separating force, maximum force and surface speed that would act on mounting shafts.

### Solution:

$$T = \frac{63000 \times P}{N}$$

$$T = \frac{63000 \times 5}{1725} = 183 \text{ in-lb}$$

Find pitch circle

$$d = \frac{Z}{P_d}$$

$$d = \frac{20 \text{ teeth}}{8 \text{ teeth/in diameter}} = 2.5 \text{ in}$$

Find transmitted force

$$F_T = \frac{2T}{d}$$

$$F_T = \frac{2 \times 183 \text{ in-lb}}{2.5 \text{ in}} = 146 \text{ lb}$$

Find separating force

$$F_R = F_T \tan \Phi$$

$$F_R = 146 \tan 20^\circ$$

$$F_R = 53 \text{ lb}$$

Find maximum force

$$F_N = \frac{F_T}{\cos \Phi}$$

$$F_N = \frac{146 \text{ lb}}{\cos 20^\circ}$$



$$F_N = 155 \text{ lb}$$

Find pitch line velocity or surface speed

$$v = .262 \times D \times \text{RPM}$$

$$v = 0.262 \times 2.5 \times 1725 = 1129 \text{ ft/min}$$

**Summarizing.....**

The fundamental equations for a gear pair are:

$$T_{in} \omega_{in} = T_{out} \omega_{out} \quad \text{----- (Power equality)}$$

$$\frac{\omega_{out}}{\omega_{in}} = \frac{r_{in}}{r_{out}} \quad \text{----- (Velocity relationship in terms of radiuses)}$$

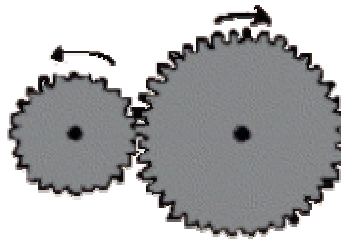
$$\frac{\omega_{out}}{\omega_{in}} = \frac{Z_{in}}{Z_{out}} \quad \text{----- (Velocity relationship in terms of number of teeth)}$$

$$\frac{T_{out}}{T_{in}} = \frac{r_{out}}{r_{in}} \quad \text{----- (Torque relationship in terms of radiuses)}$$

$$\frac{T_{out}}{T_{in}} = \frac{Z_{out}}{Z_{in}} \quad \text{----- (Torque relationship in term of number of teeth)}$$

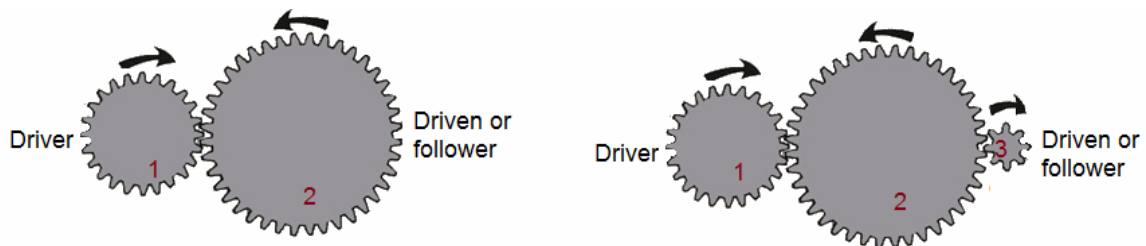
**SECTION -4****GEAR TRAINS**

A gear train is a power transmission system made up of two or more gears. The gear to which the force is first applied is called the driver and the final gear on the train to which the force is transmitted is called the driven gear. Any gears between the driver and the driven gears are called the idlers. Conventionally, the smaller gear is the **Pinion** and the larger one is the **Gear**. In most applications, the pinion is the driver; this reduces speed but increases torque.

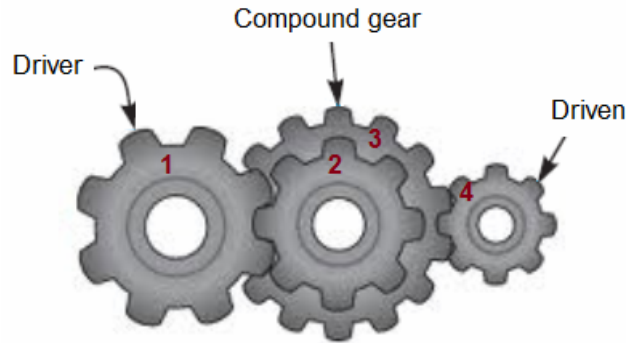
**Types of gear trains**

1. Simple gear train
2. Compound gear train
3. Planetary gear train

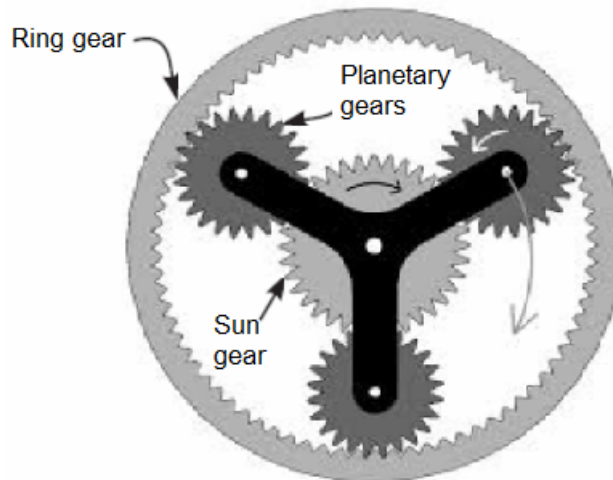
**Simple Gear Train** - Simple gear trains have only one gear per shaft. The simple gear train is used where there is a large distance to be covered between the input shaft and the output shaft.



**Compound Gear Train** - In a compound gear train at least one of the shafts in the train must hold two gears. Compound gear trains are used when large changes in speed or power output are needed and there is only a small space between the input and output shafts.



**Planetary Gear Train** - A planetary transmission system (or Epicyclic system as it is also known), consists normally of a centrally pivoted sun gear, a ring gear and several planet gears which rotate between these. This assembly concept explains the term planetary transmission, as the planet gears rotate around the sun gear as in the astronomical sense the planets rotate around our sun.



Planetary gearing or epicyclic gearing provides an efficient means to transfer high torques utilizing a compact design.

## GEAR RATIO

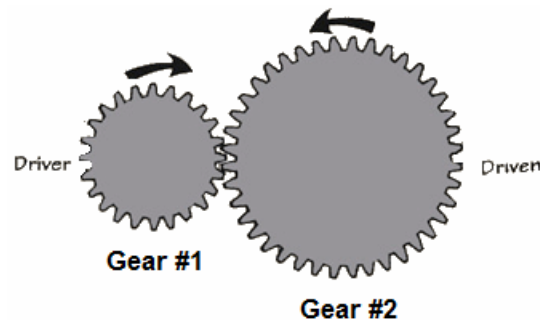
We have learnt in the previous section that ***“If two gears are in mesh, then the product of speed (revolutions) and teeth must be conserved”***. Let’s check this simple rule with a help of an example.

If you turn a gear with 6 teeth 3 times and is meshed with a second gear having 18 teeth, then the driving gear 18 teeth ( $6 \times 3$ ) will move through the meshed area. This means that the 18 teeth from the second gear also move through the meshed area. If the second gear has 18 teeth, then it only has to rotate once because  $18 \times 1 = 18$ .

Also, the second gear will be turning slower than the first because it is larger, and larger gears turn slower than smaller gears because they have more teeth.

### Gear Ratio for Simple Gear Train

Consider a simple gear train shown below. Notice that the arrows show how the gears are turning. When the driver is turning clockwise the driven gear is anti-clockwise.



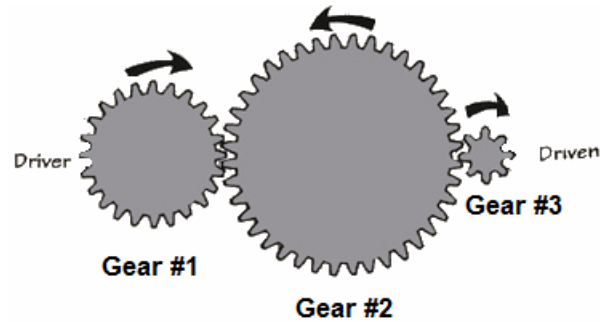
Further assume driver gear #1 has 20 teeth and rotating at 100 rpm. Find the speed of driven gear #2 having 60 teeth.

- $N_1 = 100$  rpm
- $Z_1 = 30$  teeth
- $N_2 = ?$
- $Z_2 = 60$  teeth

Solving the equation above for  $N_2$ , we have:

$$N_2 = (Z_1/Z_2) * N_1 = (30/60) * 100 = 50 \text{ rpm}$$

Let's add a third gear to the train. Assume gear 2 drives gear 3 and gear 3 has  $Z_3 = 20$  teeth. Here the driver is gear #1 and the final driven element is gear #3. Gear #2 in between is called the **idler** gear. Find the speed of driven gear #3?



Well, since gears 2 and 3 are in mesh, our conservation law says that:

$$N_2 * Z_2 = N_3 * Z_3$$

We could do the arithmetic ( $N_3 = (Z_2/Z_3) * N_2 = (60/20) * 50 = 150$  rpm) to find  $N_3$ . Or, we could note that, since both  $N_1 * Z_1$  and  $N_3 * Z_3$  are equal to  $N_2 * Z_2$ , they must be equal to each other.

$$N_1 * Z_1 = N_3 * Z_3$$

So,

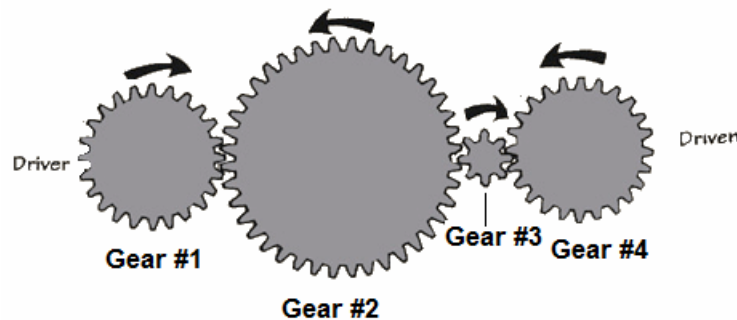
$$N_3 = (Z_1/Z_3) * N_1 = (30/20) * 100 = 150 \text{ rpm.}$$

What does this prove?

***“An idler gear between a driver and driven gear has NO effect on the overall gear ratio, regardless of how many teeth it has”.***

(Note that  $Z_2$  never entered into our computation in the last equation.)

Suppose now that we add a fourth gear with  $Z_4 = 40$  teeth to our developing gear train.



Its speed must be  $N_4 = (Z_3/Z_4) * N_3 = (20/40) * 150 = 75$  rpm. Again, by using the conservation principle, we have:

$$N_4 = (Z_1/Z_4) * N_1 = (30/40) * 100 = 75 \text{ rpm.}$$

We can continue like this indefinitely, but the two fundamental learning objectives here are:

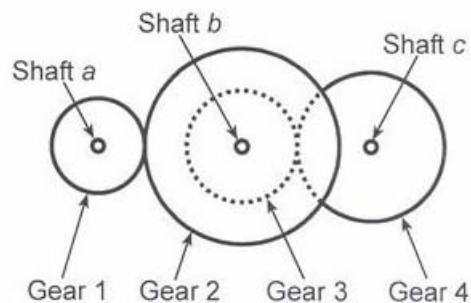
1. The number of teeth on the intermediate gears does not affect the overall velocity ratio, which is governed purely by the number of teeth on the first and last gear.
2. If the train contains an odd number of gears, the output gear will rotate in the same direction as the input gear, but if the train contains an even number of gears, the output gear will rotate opposite that of the input gear. If it is desired that the two gears and shafts rotate in the same direction, a third idler gear must be inserted between the driving gear and the driven gear. The idler revolves in a direction opposite that of the driving gear.

### Major Caveat:

Note that everything said to this point assumes a simple gear train where each of the gears in the gear train is on its own, separate shaft. Sometimes gears are 'ganged' by keying or otherwise welding them together and both gears turn as a unit on the same shaft. This arrangement is known as compound gear train and it complicates the computation of the gear ratio, to some extent.

### Compound Gear Train

The figure below shows a set of compound gears with the two gears, 2 and 3, mounted on the middle shaft b. Both of these gears will turn at the same speed because they are fastened together, i.e.  $N_b = N_2 = N_3$



When gear 1 and gear 2 are in mesh:

$$N_1 * Z_1 = N_2 * Z_2$$

It's still true that:

$$N_1 * Z_1 = N_b * Z_2$$

$$N_b = (Z_1/Z_2) * N_1$$

If gears 3 and 4 are in mesh:

$$N_b * Z_3 = N_4 * Z_4$$

Therefore,

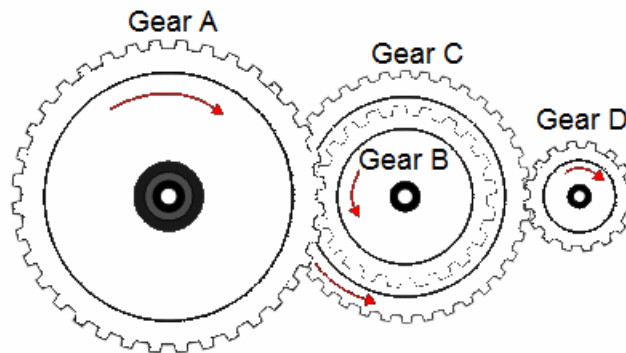
$$N_4 = (Z_3/Z_4) * N_b = (Z_3/Z_4)*(Z_1/Z_2) * N_1$$

So the end-to-end gear ratio is  $(Z_1 * Z_3) / (Z_2 * Z_4)$  and it does depend on the intermediate gears; unlike the previous case when each gear could turn on its own separate axis.

Note that the resultant gear ratio is just the product of the two separate gear ratios:  $(Z_1/Z_2)*(Z_3/Z_4)$ .

**Example:**

In the figure below, Gears B and C represent a compound gear and have the following details:



Gear A - 120 teeth

Gear B - 40 teeth

Gear C - 80 teeth

Gear D - 20 teeth

What is the output in revs/min at D, and what is the direction of rotation if Gear A rotates in a clockwise direction at 30 revs/min?

**Solution:**

When answering a question like this, split it into two parts. Treat Gears A and B as the first part of the question. Treat Gears C and D as the second part.

$$\text{Gear ratio AB} = \text{driven/driving} = 40/120 = 1/3$$

$$\text{Gear ratio CD} = \text{driven /driving} = 20/80 = 1/4$$

Since the driving Gear A rotates 30 RPM and the Gear B is smaller than Gear A, we can conclude that the RPMs for Gear B is  $30 \times 3 = 90$  RPM

Since Gears B and C represent a compound gear, they have the same rotational speed. Therefore, Gear D speed is obtained by multiplying 4 to Gear C speed of 90 RPM.

$$\text{Thus, Gear D moves at } 90 \times 4 = 360 \text{ rev/min}$$

OR

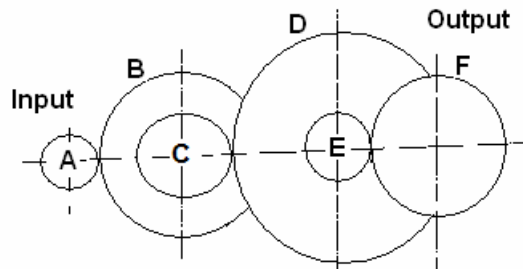
$$1/3 \times 1/4 = 1/12$$

Since Gear A moves at 30 RPM and Gear D is smaller, we multiply by 12:

$$30 \times 12 = 360 \text{ RPM}$$

**Example:**

Calculate the gear ratio for the compound chain shown below. If the input gear rotates clockwise, in which direction does the output rotate?



Gear A has 20 teeth

Gear B has 100 teeth

Gear C has 40 teeth

Gear D has 100 teeth

Gear E has 10 teeth



Gear F has 100 teeth

**Solution:**

The driving teeth are A, C and E

The driven teeth are B, D and F

$$\text{Gear ratio} = (100 \times 100 \times 100) / (20 \times 40 \times 10) = 125$$

Alternatively we can say there are three simple gear trains as follows:

$$\text{First gear GR} = 100 / 20 = 5$$

$$\text{Second chain GR} = 100 / 40 = 2.5$$

$$\text{Third chain GR} = 100 / 10 = 10$$

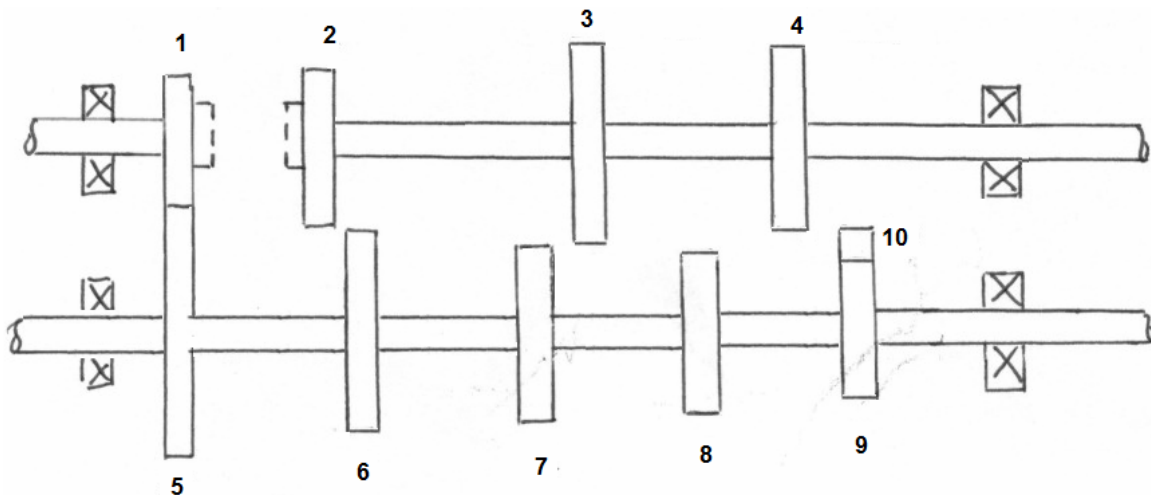
$$\text{The overall ratio} = 5 \times 2.5 \times 10 = 125$$

Each chain reverses the direction of rotation so if A is clockwise, B and C rotate anti-clockwise, so D and E rotate clockwise. The output gear F hence rotates anti-clockwise.

More complex compound gear trains can achieve high and low gear ratios in a restricted space by coupling large and small gears on the same axle. In this way gear ratios of adjacent gears can be multiplied through the gear train.

**Example:**

Suppose we look at a standard four-speed car gearbox with a reverse gear.



The following table shows the number of teeth for each gear:

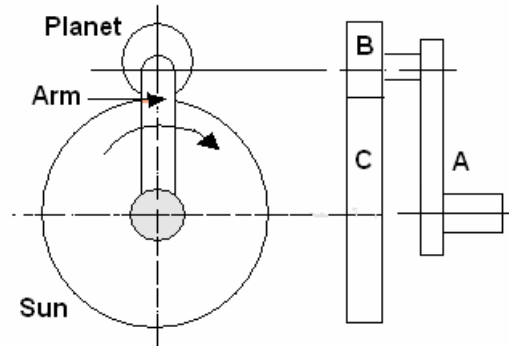
<b>Gear</b>	1	2	3	4	5	6	7	8	9	10
<b>N</b>	12	15	25	20	38	35	25	30	12	12

The table below shows the speed ratio (SR) calculations for each gear selection possible.

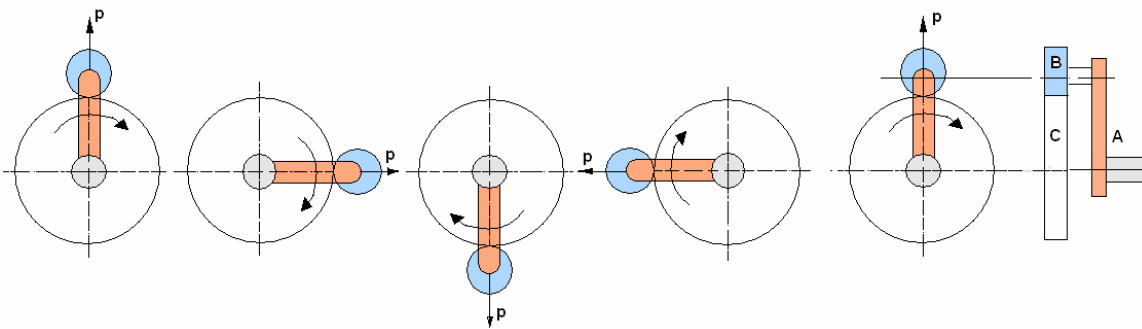
Selection	Gear Train	SR Formula	SR Formula	SR Value
1 <sup>st</sup> Gear	1,5,7,3	$\frac{N_5 \times N_3}{N_1 \times N_7}$	$\frac{38 \times 25}{12 \times 25}$	3.16
2 <sup>nd</sup> Gear	1,5,8,4	$\frac{N_5 \times N_4}{N_1 \times N_8}$	$\frac{38 \times 20}{12 \times 30}$	2.11
3 <sup>rd</sup> Gear	1,5,6,2	$\frac{N_5 \times N_2}{N_1 \times N_6}$	$\frac{38 \times 15}{12 \times 35}$	1.36
4 <sup>th</sup> Gear	1 locked with 2	None	None	1.00
Reverse	1,5, 9,10,4	$\frac{N_5 \times N_4 \times N_{10}}{N_1 \times N_9 \times N_{10}}$	$\frac{38 \times 12 \times 12}{12 \times 12 \times 12}$	-3.16

### EPICYCLIC GEAR TRAIN

In epicyclic gear train, the axis of rotation of one or more of the wheels is carried on an arm which is free to revolve about the axis of rotation of one of the other wheels in the train. The diagram shows a Gear B on the end of an arm A. Gear B meshes with Gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.



Now let's see what happens when the planet gear orbits the sun gear.



Observe point p and you will see that Gear B also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and Gear C is rotated once. B spins about its own center and the number of revolutions it makes is the ratio  $N_C / N_B$ . B will rotate by this number for every complete revolution of C.

Now consider that C is unable to rotate and the Arm A is revolved once. Gear B will revolve  $1 + (N_C / N_B)$  because of the orbit. It is the extra rotation that causes confusion. One way to get around this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

### Method 1

Suppose Gear C is fixed and the Arm A makes one revolution. Determine how many revolutions the planet Gear B makes.

Step 1 is to revolve everything once about the centre.

Step 2 is to identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions for B.

Step 3 is to simply add them up and find that the total revs of C is zero and the arm is 1.

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	$+ N_C / N_B$	-1
3	Add	1	$1 + N_C / N_B$	0

The number of revolutions made by B is  $(1+ t_C / t_B)$ . Note that if C revolves -1, then the direction of B is opposite so  $+ t_C / t_B$

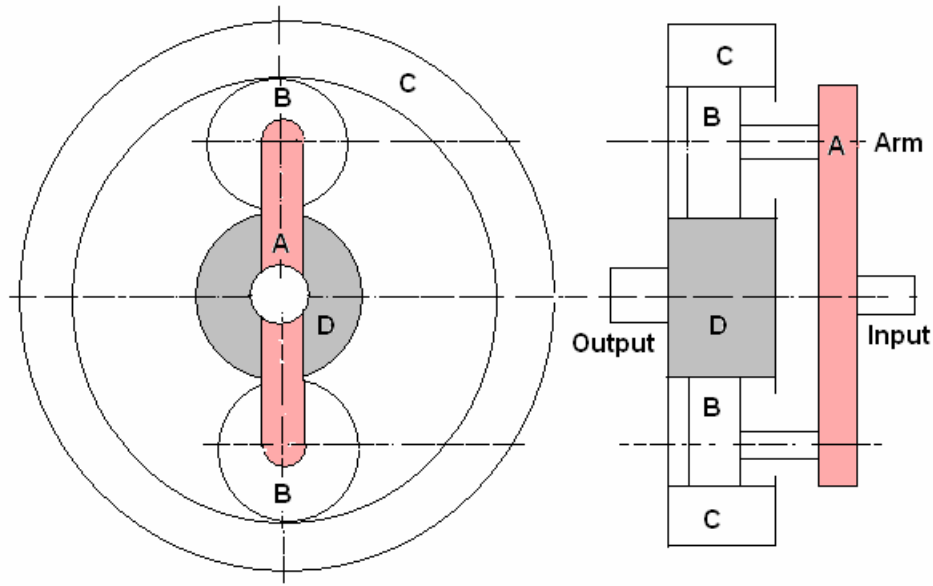
**Example:**

A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

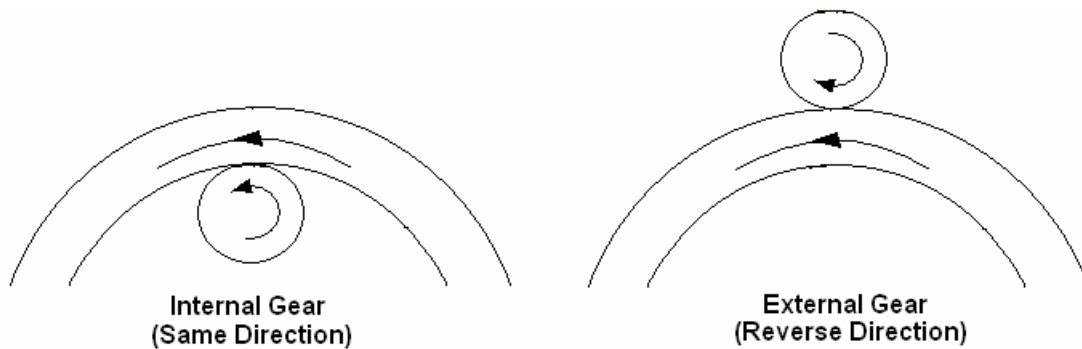
**Solution:**

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	$+ 100 / 50$	-1
3	Add	1	3	0

The design considered so far has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done in several ways.

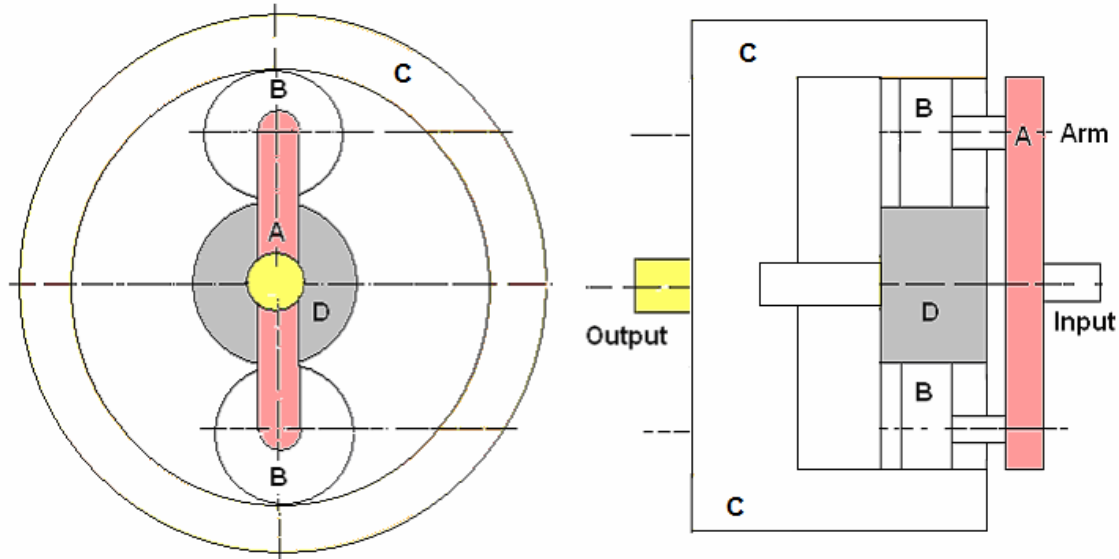


The arm is the input and Gear D is the output. Gear C is a fixed internal gear and is normally part of the outer casing of the gear box. There are normally four planet gears and the arm takes the form of a cage carrying the shafts of the planet gears. Note that the planet gear and the internal gear both rotate in the same direction.



**Method 2**

In this case the sun Gear D is fixed and the internal Gear C is made into the output.



**Example:**

An epicyclic gear box has a fixed sun Gear D and the internal Gear C is the output with 300 teeth. The planet Gears B have 30 teeth. The input is the arm/cage A. Calculate the number of teeth on the sun gear and the ratio of the gear box.

**Solution:**

$$N_C = N_D + 2 N_B$$

$$300 = N_D + 2 \times 30$$

$$N_D = 300 - 60 = 240$$

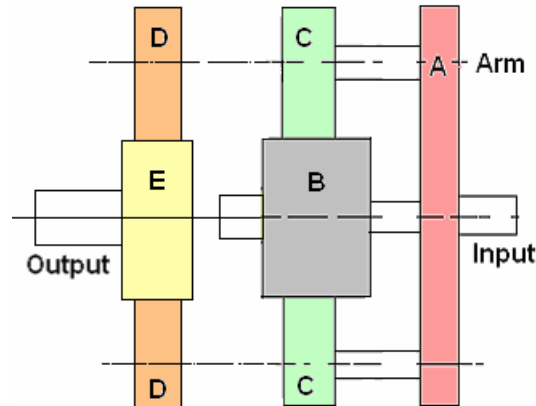
Identify that Gear D is fixed and the arm must do one revolution, so it must be D that is rotated back one revolution holding the arm stationary.

Step	Action	A	B	C	D
1	Revolve all once	1	1	1	1
2	Revolve D by -1 rev	0	240 / 30	240/300	-1
3	Add	1	9	1.8	0

The ratio  $A/C$  is then 1:1.8 and this is the gear ratio. Note that the solution would be the same if the input and output are reversed but the ratio would be 1.8.

**Method 3**

In this design a compound Gear C and D is introduced. Gear B is fixed and Gears C rotate upon it and around it. Gears C are rigidly attached to gears D and they all rotate at the same speed. Gears D mesh with the output Gear E.



**Example:**

An epicyclic gear box is shown above. Gear C has 100 teeth, B has 50, D has 50 and E has 100. Calculate the ratio of the gear box.

**Solution**

Identify that Gear B is fixed and that A must do one revolution, so it must be B that is rotated back one revolution holding A stationary.

Step	Action	A	B	C/D	E
1	Revolve all once	1	1	1	1
2	Revolve B by -1 rev	0	-1	$\frac{1}{2}$	$-\frac{1}{4}$
3	Add	1	0	$1\frac{1}{2}$	$\frac{3}{4}$

The ratio  $A/E$  is then  $\frac{3}{4}:1$  or 3:4

Note that the input and output may be reversed but the solution would be the same with a ratio of 4:3 instead of 3:4.



## SECTION –5 GEAR FAILURE & RELIABILITY ANALYSIS

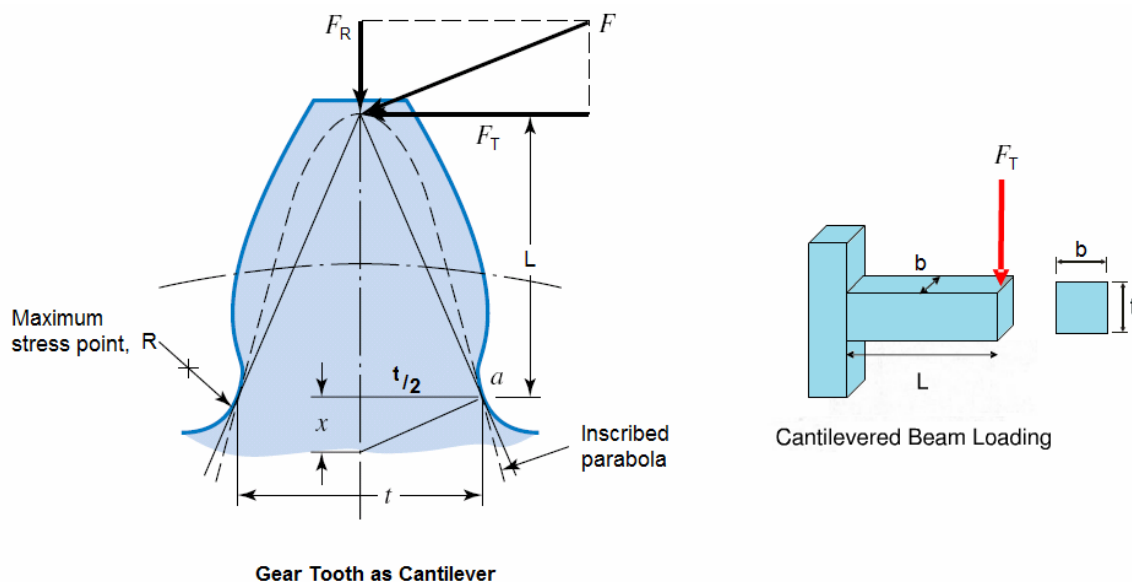
The gears are subjected to high cyclic or alternating stresses. There are three common modes of tooth failure:

1. **Pitting** is a fatigue phenomenon and occurs as a result of repeated stress cycles which lead to surface and subsurface cracks.
2. **Bending** fatigue leading to tooth breakage. It is caused by the root bending stress imposed by the transmitted load. In some cases pitting or wear may weaken the tooth to the extent that breakage occurs.
3. **Scuffing** is a form of surface damage on the tooth flanks, which occurs when the lubricant film fails allowing metal to metal contact.

### Bending Stress Calculation

Gear overload or cyclic stressing of the gear tooth at the root beyond the endurance limit of the material causes bending fatigue and eventually a crack originating in the root section of the gear tooth and then the tearing away of the tooth or part of the tooth.

The bending stress calculation is predicted by Lewis equation, which is crucial in determining the tooth width. The equation considers the gear tooth to be a cantilevered beam and uses the bending of cantilever beam to simulate the bending stress acting on the gear.



The figure above shows a cantilever of cross-sectional dimensions  $b$  and  $t$ , having a length  $L$  and a force (load)  $F_T$ , uniformly distributed across the face width  $b$ . The maximum bending stress at the base of the gear tooth is given by:

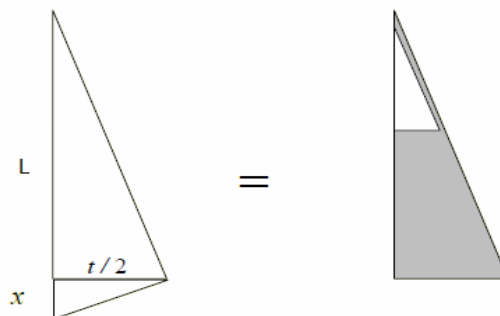
$$S = \frac{Mc}{I} = \frac{(F_T L) \frac{t}{2}}{b \frac{t^3}{12}} = \frac{6 F_T L}{b t^2} \quad \text{Eq. 16}$$

Where

- $S$  is the maximum bending stress at the base of the gear tooth, (psi)
- $M$  = Maximum bending moment =  $F_t * L$
- $c$  = Half thickness of the tooth ( $t$ ) =  $t / 2$
- $I$  = Moment inertia =  $bt^3/12$
- $F_T$  = Tangential load acting at the tooth
- $L$  = Length of the tooth
- $t$  = Gear tooth thickness at base
- $b$  = Width of gear face.

The maximum stress in a gear tooth occurs at point R as shown in the figure above. Using the similarity of triangles, we can write:

$$\frac{t/2}{x} = \frac{L}{t/2}$$



This implies:

$$L = \frac{t^2}{4x}$$

Substituting L in equation Eq. 16 above, we find

$$S = \frac{6F_T t^2}{b t^2 4x} = \frac{3 F_T}{2 b x} \quad \text{Eq.17}$$

Multiplying the numerator and denominator in Eq. 17 by the diametral pitch,  $P_d$

$$S = \frac{3 F_T}{2 b x} = \frac{3 F_T}{2 b x} \frac{P_d}{P_d}$$

Or

$$S = \left[ \frac{F_T}{b} \right] \left[ \frac{P_d}{Y} \right] \quad \text{----- Eq. 3}$$

Where, Y is the form factor and is a function of the pressure angle and number of teeth.

$$Y = \frac{2x}{3} P_d$$

The value of Y is available as in the form of a table or graph.

**Outline Factor (Y) for use with Diametral Pitch**

No. of Teeth	14½° PA Involute	20° PA Involute	No. of Teeth	14½° PA Involute	20° PA Involute	No. of Teeth	14½° PA Involute	20° PA Involute
10	.176	.201	20	.283	.320	40	.336	.389
11	.192	.226	21	.289	.326	45	.340	.399
12	.210	.245	22	.292	.330	50	.346	.408
13	.223	.264	23	.296	.333	60	.355	.421
14	.235	.276	24	.302	.337	70	.360	.429
15	.245	.289	25	.305	.340	80	.363	.436
16	.255	.295	26	.308	.344	90	.366	.442

17	.264	.302	28	.314	.352	100	.375	.446
18	.270	.308	30	.318	.358	150	.378	.458
19	.277	.314	35	.327	.373	Rack	.390	.484

Using the Lewis equation, one can determine the value of the tooth width, b, by substituting the maximum allowable stress value of material as follows:

$$S = \left[ \frac{F_T}{b} \right] \left[ \frac{P_d}{Y} \right]$$

**Average (S) values in pounds per square inch**

Material	S (kpsi)
Steel (Normalized, 140 Bhn)	19 - 25
Steel, Q & T, 180 Bhn	25 – 33
Steel, Q & T, 300 Bhn	36 – 47
Steel, Q & T, 400 Bhn	42 - 56
Steel (Case carburized)	55 - 65
Steel (Nitrided)	34 -45
Cast Iron (AGMA Grade 30, 175 Bhn)	8.5
Cast Iron (AGMA Grade 40, 200 Bhn)	13
Bronze (AGMA 2C, Sand cast 40 ksi)	5.7
Nonmetallic Nylon	6000

In design of gears, the pinion is made harder than the gear. Why?

The Lewis equation indicates that the tooth bending stresses vary (1) directly with load  $F_T$ , (2) inversely with tooth width b, (3) directly as a diametral pitch or inversely with the

tooth size (note that gear teeth size varies inversely with diametral pitch), and (4) inversely with tooth shape factor Y.

$$S = \left[ \frac{F_T}{b} \right] \left[ \frac{P_d}{Y} \right]$$

It can be observed that 'Pd' and 'b' are the same for pinion as well as for gear in a gear pair. When different materials are used, the product Y and S decides the weaker between the pinion and gear.

The Lewis form factor Y is dependent on the number of teeth and therefore Y will always be less for pinion compared to gear. Therefore, when the same material is used for pinion and gear, the pinion is always weaker than the gear. Pinions should therefore be made approximately 40 BHN harder than their mating wheel to even out the life of the two parts with respect to fatigue and wear.

#### Limitations of Lewis Equation

Lewis equation considers only the static loading and doesn't take into account the dynamics of meshing teeth. It assumes that at any time only one pair of teeth is in contact and takes the total load. The effect of stress concentration or the effect of radial component ( $F_R$ ), which induces compressive stresses, is neglected. Further, it assumes that the tangential component ( $F_T$ ) is uniformly distributed over the face width of the gear (note that this is only possible when the gears are rigid and accurately machined).

The Lewis stress formula must therefore be modified to account for the varying situations like stress concentration and geometry of the tooth.

#### A More Realistic Approach - AGMA Strength Equation

The AGMA approach, while based on the idealized Lewis equation, involves an extensive list of empirical adjustment factors to account for the influence of various manufacturing, assembly, geometric, loading, and material variability's. While incorporating all of these factors, the Lewis strength equation will be modified as follows:

$$S = \left[ \frac{F_T}{b} \right] \left[ \frac{P_d}{J} \right] \left[ \frac{K_a K_s K_m}{K_v} \right]$$

Where:

- $S_b$  bending stress, (psi)

- $K_a$  application factor
- $K_s$  size factor
- $K_m$  load distribution factor
- $K_v$  dynamic load factor or velocity factor
- $P_d$  is the diametral pitch
- $J$  geometry factor

**Note!**

Dynamic factor  $K_v$  has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards it was less than 1.0. Care must be taken in referring to work done prior to this change in the standards.

**Application Factor,  $K_a$ ,** accounts for non-uniform transmitted loads. It allows for the non-uniformity of input and/or output torque inherent in the machinery connected to the gears. Some of the pertinent application influences include type of load, type of prime mover, acceleration rates, vibration, shock, and momentary overloads. Suggested factors are tabulated below:

**Suggested Application Factors  $K_a$  for Reduction Gears**

Driven Machinery		Driving Machinery		
		Uniform Electric motor, Steam turbine Gas turbine	Light Shocks Multi- cylinder combustion engine	Heavy Shocks Single cylinder combustion engine
Uniform	Generator, Belt conveyor, Light Elevator, Electric hoist, Machine tool feed drive, Ventilator, Turbo- blower, Turbo-compressor, Mixer (constant density)	1	1.25	1.5

Medium Shocks	Machine tool main drive, Heavy elevator, Crane turning gears, Mine ventilator, Mixer (variable density), Multi-cylinder, Piston pump, Feed pump	1.25	1.5	1.75
Heavy Shocks	Press, Shear, Rolling mill drive, Heavy centrifuge, Heavy feed pump, Pug mill, Power shovel, Rotary drilling apparatus, Briquette press	1.75	2	2.25

**Size Factor,  $K_s$**  - reflects the influence of non-homogeneous materials. The size factor  $K_s$  corrects the stress calculation to account for the known fact that larger parts are more prone to fail.

- Usually “1” is used.
- For large teeth, 1.25 to 1.5 would be used.

**Load Distribution or Mount Factor,  $K_m$**  is intended to account for distribution of load across face.

**Load Distribution Factors  $K_m$  for Spur and Helical Gears**

Condition of support	Face width, b, (inches)							
	≤2		6		9		16	
	Spur	Helical	Spur	Helical	Spur	Helical	Spur	Helical
Accurate mounting, low bearing clearances, minimum elastic deflection, precision gears	1.3	1.2	1.4	1.3	1.5	1.4	1.8	1.7

Less rigid mountings, less accurate gears, contact across full face	1.6	1.5	1.7	1.6	1.8	1.7	2.0	2.0
Accurate and mounting such that less than full face contact exists			Over 2.0					

**Dynamic Effect (Kv)** is intended to correct for the effects of the speed of rotation and the degree of precision on gear accuracy. Barth first expressed the velocity factor in terms of the current AGMA standards; they are represented as:

$$K_v = \frac{600 + v}{600} \quad (\text{Cast iron, cast profile})$$

$$K_v = \frac{1200 + v}{1200} \quad (\text{Cut or milled profile})$$

Where, v is the pitch-line velocity, in feet per minute.

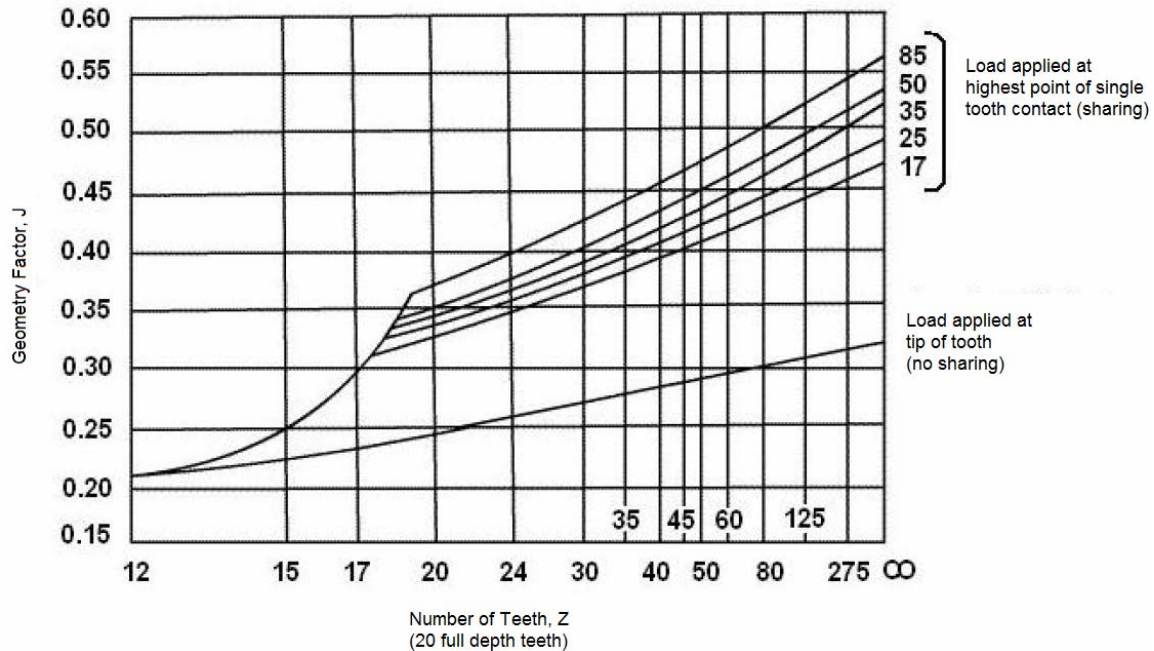
$$K_v = \frac{50 + \sqrt{v}}{50} \quad (\text{Hobbed or shaped profile})$$

$$K_v = \sqrt{\frac{78 + \sqrt{v}}{78}} \quad (\text{Shaved or ground profile})$$

These equations form the basis for the AGMA approach to the bending strength of gear teeth. They are in general use for estimating the capacity of gear drives when life and reliability are not important considerations. The equations can be useful in obtaining a preliminary estimate of gear sizes needed for various applications.

**Geometry Factor, J** is a modification of the form factor Y to account for three more influences: stress concentration, load sharing between the teeth, and changing the load application point to the highest point of single-tooth contact. This factor depends on the shape of the tooth and the distance from the tooth root to the highest point of single-tooth contact. The value of spur gears with 20 pressure angle and full-depth teeth is found from the graph below:





The transmitted tooth load  $F_T$  is equal to the torque divided by the pitch radius for spur and helical gears.

### Gears Reliability

The previous paragraphs have provided an insight into the specific characteristics and failure modes of the gears. The performance and the useful life of a machine is also governed by the reliability of gears. The gears manufactured to high standards of American Gear Manufacturer's Association (AGMA) provide extreme reliability.

Under normal circumstances, reliability is evaluated as part of the reliability factor ( $K_R$ ) which accounts for the effect of the normal statistical distribution of failures found in material testing. Gear teeth designed to AGMA standards are based upon a statistical probability of fewer than one failure in 100. If your experience show that  $K_R$  of 1.25 has given satisfactory service in the past with normal maintenance, the identical gear drive should be used on new purchases. If, on the other hand, you have a new, yet-to-be-proved application, a more appropriate reliability factor may be one failure in 1,000 or even one in 10,000.

For any given load ( $L$ ), the life and survival rate (reliability) may be correlated through:

$$L = S * \text{life factor } (K_L) / \text{reliability factor } (K_R)$$

Where:

- $L$  = load
- $S$  = maximum allowable stress for the property of steel
- $K_R$  = the *reliability factor*, caters for survival rates other than 99%. Since the survival contours are essentially parallel to one another on a logarithmic scale, then simple multiplication factors enable load correlation, as indicated in the table below:

RELIABILITY FACTOR	% Survival	$K_R$
fewer than one failure in 10,000	99.99	1.50
fewer than one failure in 1,000	99.9	1.25
fewer than one failure in 100	99	1.00
fewer than one failure in 10	90	0.85

- $K_L$  = the *life factor* ( $K_L$  for bending,  $C_L$  for pitting) caters for lives other than  $10^7$  cycles. Since the load-life diagrams for all the steels considered are of the same shape essentially, normalizing by the allowable stress will result in a unique  $K_L$  (or  $C_L$ ) vs. life curve for all steels.

## GEAR MATERIALS

When specifying gear materials, properties such as resistance to wear, good fatigue strength as well as a low coefficient of friction are desirable. **Alloy steels** are most commonly used in manufacture of gears. They offer high strength and a wide range of heat treatment properties. The material composition below indicates how properties vary with commonly used alloy materials:

1. **Nickel** - Increases hardness and strength.
2. **Chromium** - Increases hardness and strength but the loss of ductility is greater. It refines the grain and imparts a greater depth of hardness. It has high degree of wear resistance.

3. **Manganese** - It gives greater strength and a high degree of toughness than chromium.
4. **Vanadium** - The hardness penetration is greatest. The loss of ductility is also more than any other alloys.
5. **Molybdenum** - Increases strength without affecting the ductility.
6. **Chrome - Nickel Steels** - The combination of the two alloying elements, chromium and nickel, adds the beneficial qualities of both. Gears made from 300 series stainless steel, containing 18% chromium and 8% nickel, are essentially nonmagnetic and cannot be hardened by heat treatment. They are recommended for low torque applications as their mechanical properties and resistance is low.

The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness. It is NOT essential for both pinion and wheel gears to be of the same material. As the smaller gear will have to rotate more turns than the larger gear, it is more prone to wear and fatigue. It is common, therefore, to choose a material with improved properties for the pinion to give a gear pair with a near matching strength and durability.

#### **Simplified Rules of Thumb for Design of Gears**

- The face width of spur gears should be 3 to 5 times the circular pitch.
- If possible, the ratio of face width to pitch diameter should be kept small ( $<2$ ) for less sensitivity to misalignment and uneven load distribution due to load-sensitive deflections.
- Increasing the face width will decrease wear and fatigue, but the dynamic load will increase.
- A coarser tooth (smaller diametral pitch) will improve the fatigue performance but not the wear performance. For a given tooth design, only a harder material will improve the wear.
- Larger pitch diameters have lower tangential forces and lower dynamic loadings.
- Decreasing the tooth error decreases the dynamic loading.

## **SECTION -6                      HOW TO SPECIFY & SELECT GEAR DRIVES**

The choice of a gear drive depends on the application, its environment and the physical constraints of the system. The gearbox geometry is defined by four parameters which are determined by the characteristics of the driving and driven machinery:

1. Horsepower transmitted
2. Speed of the driving gear
3. Ratio required (reduction or increasing)
4. Arrangement of shafting

Gears can either be obtained as standard components from a manufacturer's catalogue or alternatively specially designed and manufactured. Gear catalogues tend to display only geometric and materials data of stock gears rather than specific operational information. This is because functional behavior will vary with an application and it is not feasible to give comprehensive data covering all operational conditions within a catalogue. The operational factors for deciding the type of the gear are:

1. Shaft orientation
2. Operating environment
3. Speed ratio
4. Nature of load
5. Service factor
6. Gear drive rating
7. Overhung load
8. Gear lubrication
9. Gear materials and heat treatment
10. Efficiency
11. Noise considerations
12. Maximum speed
13. Power transmission capacity
14. Costs

All must be carefully evaluated to make the right decision.

### **Shaft Orientation:**

Various shaft arrangements are possible. Use:

- Spur & Helical Gears, when the shafts are parallel
- Bevel Gears, when the shafts intersect at right angles, and
- Worm & Worm Gears, when the axes of the shaft are perpendicular and not intersecting.
- As a special case, when the axes of the two shafts are neither intersecting nor perpendicular, crossed helical gears are employed.

### **Operating Environment:**

Check your application and the operating environment.

Contact seals should be used on input and output shafts when the unit operates in dusty environments or where water is splashed around the unit. In atmospheres laden with abrasive dust or in areas hosed down with water under pressure, two contact seals may be required on each shaft. Typically, an enclosure around the gears with oil lubrication is the preferred design, but grease-lubricated open gears can be used in relatively clean environments.

Moisture or high humidity is another concern. A key instance of this is a food processing environment requiring washdowns. In such cases, consider reducers with special epoxy coatings, external shaft seals, and stainless steel shaft extensions and hardware.

### **Speed Ratio:**

You arrive at the specific gear ration by dividing the motor full-load speed to the revolutions per minute (RPM) of the driven equipment. Theoretically, there is no limit to the speed ratio that can be designed into a single reduction gearbox, but there is an approximate ratio for each type of gear above where the materials are not being used economically. These ratios are:

Type	Normal Ratio Range
Spur	1:1 to 6:1
Straight Bevel	3:2 to 5:1
Spiral Bevel	3:2 to 4:1
Worm	5:1 to 75:1
Hypoid	10:1 to 200:1
Helical	3:2 to 10:1
Cycloid	10:1 to 100:1

**Important!**

- For high speed reduction, two-stage or three-stage construction should be used.
- For applications with variable frequency drives, exact gear ratios become less important. In such cases, it is best to select the manufacturer’s standard ratios, which is less expensive. Variable or multi-speed applications, however, require special considerations to provide adequate splash lubrication at the slowest speed, without excessive heating or churning at the higher speed.

**Nature of load:**

A gear drive is one part of a power system which has certain load characteristics peculiar to the specific application. The operating characteristics fall into two load categories: constant torque or constant horsepower.

- Constant torque occurs when load demand varies proportionally with a change in speed. Examples are conveyors, stokers, and reciprocating compressors. Gear drives are basically constant torque machines requiring no selection modifications.
- Constant horsepower implies load demand is constant regardless of speed. Examples are lathes, boring mills, radial drill presses, etc. The gear drive must

be selected for the slowest speed at which the motor will deliver its rated horsepower capacity.

The type of load on the gear drive also depends on the operational characteristics of the prime mover. Electric motors and turbines produce relatively smooth operation, whereas an internal combustion engine does not afford so smooth a load.

**Service factor (SF):**

Service factors are used to take into consideration intangible operating conditions such as misalignments, vibrations, transient loads and shocks.

The actual horsepower is multiplied by the service factor to obtain an equivalent horsepower, and the gear unit selected must have a rating equal to or greater than the equivalent horsepower. Typically, this service rating is determined by multiplying the required horsepower by the appropriate service factor based on the equipment, duty cycle, and type of prime mover. A SF value between 1.25 and 2.0 is typically chosen and then multiplied by the motor nameplate power to establish that required by the driven equipment.

Type of Load	Service Factors		
	Intermittent or 3hrs per day	8 -10hrs per day	Continuous 24hrs per day
Uniform	0.80	1.00	1.25
Light shock	1.00	1.25	1.50
Medium shock	1.25	1.50	1.80
Heavy shock	1.50	1.80	2.00

**Important!**

Unless otherwise designated, assume manufacturer’s ratings are based on an AGMA-defined service factor of 1.0, meaning continuous operation for 10 hours per day or less

with no recurring shock loads. If conditions differ from this, input horsepower and torque ratings must be adjusted for specific applications. A higher SF, or larger gear drive size, should be selected when peak running loads are substantially greater than normal operating loads.

### **Gear Drive Rating:**

The function of power transmission gear drive is to reliably transmit torque and rotary motion between a prime mover and a driven piece of equipment at acceptable levels of noise, vibration and temperature. For every gear drive there is a mechanical rating and thermal rating.

- **Mechanical Rating** – The mechanical rating indicates the load the gear drive can transmit based on stress and wear considerations. The relationship between gear life and load is related to the service factor (SF); for example, if SF is increased by 30 percent, the gear tooth life will increase 10 times.
- **Thermal Rating** – The thermal rating specifies the power that can be transmitted continuously for 3 hours or more without exceeding a specified temperature rise above ambient. Typically, enclosed drives operate at a temperature rise of 70 to 100°F above ambient temperature. The maximum acceptable oil-sump temperature is 200°F per AGMA. Exceeding these guidelines can shorten the life. Be sure to provide adequate air space around for heat dissipation, or adjust the oil viscosity where temperatures vary widely. For low-temperature operation, the oil should have a pour-point lower than that of the extreme minimum temperature encountered. Also, the oil may require pre-heating under extremely cold starting conditions.

### **Overhung Load:**

This is a force applied at right angles to a shaft beyond the shaft's outermost bearing. Too much overhung load can cause bearing or shaft failure. Unless otherwise stated, a gearbox manufacturer's overhung load maximums are rated with no shaft attachments such as sheaves or sprockets. The American Gear Manufacturers Association provides factors, commonly called "K" factors, for various shaft attachments by which the



manufacturer's maximum should be reduced. Overhung load can be eased by locating a sheave or sprocket as close to the reducer bearing as possible. In cases of extreme overhung load, an additional outboard bearing may be required.

The following formula can be used to calculate overhung load (OHL):

$$\text{OHL (pounds)} = T \times K / R$$

Where,

- T= Torque (inch-pound)
- K = load factor constant equals 1.00 for a chain and sprocket, 1.25 for a gear, and 1.5 for a pulley and a v-belt.
- R = Radius of gear

#### **Gear Lubrication:**

The purpose of the gearbox lubrication system is to provide an oil film at the contacting surfaces of working components and absorb heat generated in the gearbox so that component temperatures are not excessive. The majority of the oil flow is required for the cooling function.

As stated in the previous paragraph, AGMA thermal ratings are based on a maximum oil sump temperature of 200°F. In turbo-machinery applications, the thermal rating is usually less than the mechanical rating and an external cooling system is required. When designing a lubrication system, the initial step is to estimate the oil flow to the components and the gearbox efficiency.

The temperature rise across the gearbox can then be calculated:

$$\Delta t = Q / m C_p$$

Where:

- $\Delta t$  = Temperature rise, °F
- m = Flow, Lbs/min (Note: 1 GPM = 7.5 Lbs/min)
- Q = Heat, BTU/min (Note: Q = HP x 42.4)
- $C_p$  = Specific Heat = .5 BTU/Lb, °F

For example, a gearbox transmitting 1000 HP with 98% efficiency will reject 20 HP or 848 BTU/min of heat to the oil. If the gearbox flow is 20 GPM or 150 Lbs/min, the temperature rise across the gearbox will be 11°F. Typical operating temperatures for turbo-machinery gearboxes are around 130°F with a rise of 30°F. These values are for mineral oils; synthetic oils may operate at higher temperature levels.

The choice of lubricant depends on operating conditions:

- At peripheral speeds up to 3500 fpm, it is preferable to use a lubricant with a low viscosity to avoid excessive churning of the fluid and to facilitate splash lubrication.
- For very high gear forces, lubricants with a greater viscosity are used, and for faster gear speeds a pressurized feed system may be necessary.

Most turbo-machinery gearboxes utilize an AGMA type 1 or 2 lubricant.

The amount of oil flow supplied to a gear mesh is generally based on experience and experimental data. A rule of thumb sometimes used is .02 Lbs/min/HP.

### **Important!**

Gear drives are best driven at input speeds common in industrial electric motors, typically 1200, 1800 or 2500 RPM. This provides sufficient “splash” for the reducer’s lubrication system, but not so much as to cause oil “churning.” For input speeds under 900 RPM or above 3000 RPM, consult the manufacturer. Alternative lubricants may be suggested. Note for peripheral speeds less than 100 fpm grease will suffice.

### **Gear Materials and Heat Treatment:**

Proper choice of gear material is probably the most important factor in the successful operation of a gear set. The material for any gear is selected based on:

- The working condition, i.e. power, speed and torque to be transmitted;
- Working environment, i.e., temperature, vibration, chemical, etc.
- Ease of manufacture; and
- Overall cost of material and manufacture.

In choosing a gear material the tooth hardness and type of heat treatment to achieve that hardness must be considered.

Gears, case hardened by “**Carburizing**”, provide the best metallurgical characteristics combining good case structure with reasonable ductility. Carburizing produces the “strongest” gear providing bending and pitting fatigue resistance and an excellent wear surface. A disadvantage of carburizing is that gear teeth distort during the heat treatment and, in order to obtain high precision, grinding after hardening is required. Surface hardness attained by carburizing is in the order of Rc 55-62. For critical applications Rc 60 minimum should be specified. The best carburizing steel to achieve high load carrying capacity is SAE 9310 (AMS 6260 or AMS 6265). Other carburizing steels used include SAE 8620, 4620, 4320 and 3310.

The “**Nitriding**” process is used to case harden gears when distortion must be held to a minimum. Often the gears are finish cut and then nitrided, eliminating the grinding requirement. Nitrided gears do not have the bending and pitting fatigue resistance of carburized gears but do provide a hard, wear resistant case. With nitriding steel such as AMS 6475 (nitralloy N) or AMS 6470 (nitralloy 135), case hardnesses of RC 65-70 can be achieved. Steels such as SAE 4140 and 4340 are nitrided to hardnesses of 320 to 380 BHN.

Through hardened gears in turbo-machinery, units are generally in the 300-400 BHN hardness range. Typical through hardening steels are SAE 4140 and 4340.

Often, combinations of pinion and gear materials and heat treatments are used. For instance, if the gear is very large it may not be practical to harden, and the gear set might consist of a carburized and ground pinion driving a through hardened gear.

### **Efficiency:**

The losses in a gear transmission system can be divided into two categories:

1. **Load losses:** which are proportional to the load transmitted, and are mainly due to tooth’s friction.
2. **No-load losses:** which are constant for a given operating speed and temperature, and to the churning of the lubricant, oil seal friction, etc.

The tooth losses of helical, spur and bevel gears are small, since their tooth actions are predominantly rolling. The no-load losses can vary from a small proportion to as much as 80% depending on the gear peripheral speeds and the types of bearings used. A good practical guide to the efficiency of this class of gear, mounted in an anti-friction bearing and lubricated with oil, is to allow 1% loss per gear mesh when transmitting full load. On the other hand, at part load, the efficiency will be lower since the fixed no-load losses are a higher percentage of the smaller total load.

Worm gear tooth action, on the other hand, is predominantly sliding. Therefore, the tooth losses are higher comparing with helical, spur, and bevel gears. They depend mainly on the load angle of the worm and the coefficient of friction at the contact, which varies widely with speed.

**Gear Efficiency Comparison Table**

Type	Normal Ratio Range	Efficiency Range
Spur	1:1 to 6:1	94-98%
Straight Bevel	3:2 to 5:1	93-97%
Spiral Bevel	3:2 to 4:1	95-99%
Worm	5:1 to 75:1	50-90%
Helical	3:2 to 10:1	94-98%
Hypoid	10:1 to 200:1	80-95%

#### **Noise Considerations:**

The most significant factor for noise level is the total contact ratio. Gear noise may be reduced by increasing either the profile or face contact ratio. Studies indicate that high-contact-ratio spur gears (with a 58 percent increase in profile contact ratio) showed an average noise reduction of about 2 dB over standard gears. The same applies to helical and other gear types.

The other factors are the speed and the gear tooth profile. The involute tooth profile is typically quieter than their non-involute counterparts.

#### **Maximum Speed:**

A high speed unit is defined as operating with a pinion speed of 3600 revolutions per minute and higher, or pitch line velocities of 5000 feet per minute and higher ( $V = .262 * \text{Pitch diameter} * \text{Revolutions per minute}$ ).

Speed limits are generally based on a maximum pitch line velocity and sliding velocity in case of worm gears. Approximate maximum pitch line speeds for various types of high precision gears are:

- Spur = 20,000 fpm
- Helical = 40,000 fpm
- Straight Bevel = 10,000 fpm
- Spiral Bevel = 25,000 fpm
- Worm = 14,000 fpm

Standard catalog gear units are listed to approximately 20,000 feet per minute. Applications exceeding this speed must be considered special and exceptional care must be taken in their design and manufacture.

Many of the speed limitations are concerned with the acceptable noise level and what is acceptable on one application may be unacceptable for the other. Therefore, the figures should not be taken as strict rules.

**Note** - The noise increases sharply with the increase in peripheral speed and to a lesser extent with the increase in tooth load.

**Power Transmission Capacity:**

Power capacity of gears is limited by resistance to two forms of failure: 1) one being tooth's surface fatigue, (pitting), which sometimes is referred to as wear rating, and 2) the other one is the tooth's bending fatigue, which is referred to as strength rating. Current manufacturing capacity, known materials, and method of lubrication limit the maximum power that can be transmitted through gears of different types. Approximate maximum powers are:

- Spur Gear = 25,000 HP
- Helical Gear = 25,000 HP
- Spiral bevel = 2,950 HP
- Worm Gear = 1,000 HP

These values vary with ratio and are only intended to give practical guidance on what is available commercially.

**Costs:**

- Spur gears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them.
- Single reduction worm gear units of high-speed ratio have significantly higher power losses than other types, but set against this are the low initial cost, high reliability due to the small number of components, and the low noise level.
- Helical gear units have low power losses, but have higher initial cost, often requiring two or three reduction stages against it making it slightly less reliable with a higher noise level.
- Generally gear costs increase with module size (i.e. tooth size and hence gear diameter) and gear type (due to manufacturing complexity). Typically a helical gear with a metric module of, say, 3 will be about two to three times more expensive than one, with the same number of teeth, with a module of 2.
- Also, spur gears will be less expensive than comparably sized helical gears, and in turn, helical gears will cost less than internal, double helical and skew helical gear types.

**Important!**

Larger diameter gears have larger bore sizes and so require larger shafts and possibly bearings also; this adds further to the overall cost.

**CODES & STANDARDS**

Gears can either be obtained as standard components from a manufacturer's catalogue or alternatively specially designed and manufactured. American gear manufacturer's association manuals, AGMA 2001-C95 or AGMA-2101-C95 provide guidelines to the selection of gears. Other relevant standards are:

- ISO 6336-1 to 5:1996
- British BS 4582
- German DIN 867 & 3963

## **Summary**

In transmitting rotary motion from one shaft to another, gears provide a positive ratio type of drive. Gears are of several categories and can be combined in a multitude of ways, some of which are meshing circular spur gears, rack and pinion spur gears, and worm gears. Helical and herringbone gears utilize curved teeth for efficient, high-capacity power transmission. Worm gears, driven by worms, transmit motion between non-intersecting right-angle axes.

When two gears are connected, they rotate in opposite directions. The only way that the input and output shafts of a gear pair can be made to rotate in the same sense is by interposition of an odd number of intermediate gears. Such a gear train is called a simple train. If there is no power flow through the shaft of an intermediate gear, then it is an idler gear. The gear that does the driving is known as the driver and the other is known as the driven gear. If two gears have the same number of teeth, then one turn of driver gear causes the driven gear to turn once. When the driver gear is smaller than the driven gear, then the speed is reduced and this amplifies the torque in proportion to their teeth numbers. The pinion is the smallest gear and the larger gear is called the gear wheel.

The shape of the gear teeth is important in order to produce a smooth transfer of the motion. When the teeth action is such that the driving tooth moving at constant angular velocity produces a proportional constant velocity of the driven tooth, the action is termed a conjugate action. The teeth shape universally selected for the gear teeth is the involute profile.

One essential point for the proper meshing of the gears is that the size of the teeth on the pinion should be the same as the size of the teeth on the wheel. The module must be common to both gears. Pitch circles contact one another at the pitch point and the pinion's pitch line velocity must be identical to the wheels pitch line velocity. At the pitch point, it develops a tangential component of action-reaction due to contact between the gears.

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