Project Management with Critical Path Method Applications

Course No: B04-009
Credit: 4 PDH

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Section 1 — Background

Project management is the discipline of organizing and managing resources (e.g. people) in a way that the project is completed within defined scope, quality, time, and cost constraints. A project is a temporary and one-time endeavor undertaken to create a unique product or service, which brings about beneficial change or added value. This property of being a temporary and one-time undertaking contrasts with processes, or operations, which are permanent or semi-permanent on-going functional work to create the same product or service over and over again. The management of these two systems is often very different and requires varying technical skills and philosophy, hence requiring the development of project managers to lead projects. The primary challenge of project management is to achieve all of the project goals and objectives, while adhering to the project constraints. The secondary--and more ambitious--challenge is the optimized allocation and integration of inputs needed to meet pre-defined objectives. A project is a carefully defined set of activities that use resources (money, people, materials, energy, space, provisions, communication, motivation, etc.) to meet the pre-defined objectives.

Section 2 — Job Description

Project management is quite often the province and responsibility of an individual project manager. This individual seldom participates directly in the activities that produce the end results, but rather strives to maintain the progress and productive mutual interaction of various parties in such a way that overall risk of failure is reduced. A project manager is often a client representative and has to determine and implement the exact needs of the client, based on the knowledge of the firm they are representing. The ability to adapt to the various internal procedures of the contracting party, and to form close links with the nominated representatives, is essential in ensuring that the key issues of cost, time, quality, and above all, client satisfaction, can be achieved.

Section 3 — The Traditional Triple Constraints

Like any human undertaking, projects need to be performed and delivered under certain constraints. Traditionally, these constraints have been listed as scope, time, and cost also known as resources. These are also referred to as the Project Management Triangle, where each side represents a constraint as illustrated in Figure 1. One side of the triangle cannot be changed without impacting the others. A further refinement of the constraints separates the product 'quality' or 'performance' from the scope, and turns quality into a fourth constraint.

Figure 1 Traditional Triple Constraints

The time constraint refers to the amount of time available to complete a project. The cost constraint refers to the budgeted amount of money available for the project. The scope constraint refers to what must be done to produce the project's end results. These three constraints are often competing constraints: increased scope typically means increased time and increased cost, a tight time constraint could mean increased costs and reduced scope, and a tight budget could mean increased time and reduced scope. The discipline of project management is about providing the tools and techniques that enable the project team (not just the project manager) to organize their work to meet these constraints. Another approach to project management is to consider the three constraints as finance, time, and human resources. If a job needs to finish in a shorter time, more people can be thrown at the problem, which in turn will raise the cost of the project; unless by doing this task quicker, costs are reduced elsewhere in the project by an equal amount.

Section 4 — CPM-Critical Path Method

Overview

In 1957, Dupont developed a project management method designed to address the challenge of shutting down chemical plants for maintenance and then re-starting the plants once the maintenance had been completed. Given the complexity of the process, he developed the Critical Path Method (CPM) for managing such projects.

CPM provides the following benefits:

• Provides a graphical view of the project.

• Predicts the time required to complete the project.

• Shows which activities are critical to maintaining the schedule and which are not.

Project Planning

Project planning is a part of project management, which relates to the use of schedules such as Gantt charts to plan and subsequently report progress within the project environment. Initially, the project scope is defined as the appropriate methods for completing the project are determined. Following this step, the durations for the various tasks necessary to complete the work are listed and grouped into a work breakdown structure (WBS). The logical dependencies between tasks are defined using an activity network diagram that enables the identification of the critical path. Float or slack time in the schedule can be calculated using the appropriate equations. Then the necessary resources can be estimated and costs for each activity can be allocated to each resource, thus, estimating the total direct project cost. At this stage, the project plan may be optimized to achieve the appropriate balance between resource usage and project duration to comply with the project objectives. Once established and agreed upon, the plan becomes what is known as the baseline. Progress will be measured against the baseline throughout the life of the project. Analyzing progress compared to the baseline is known as earned value management.

Steps in CPM Project Planning

The following steps should be followed in the project planning stage:

1.   Specify the individual activities.

2.   Determine the sequence of the activities.

3.   Draw the network diagram.

4.   Estimate the completion time for each activity.

5.   Identify the critical path (longest path through the network).

6.   Schedule, allocate, and level the resources when the availability is limited.

7.   Update the CPM diagram as the project progresses.

8.   Control (Crash) the CPM diagram (assuming fast-track project or as a result of updating.

Every above-listed step is thoroughly elaborated on in the coming sections with detailed examples. These examples are related to each other.

From the work breakdown structure (WBS) and for the bill of quantity (BOQ), a listing can be made of all activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.

Example 1

To illustrate the use and importance of project management in any project, consider the following network that consists of ten activities, A through J. Specify the activities.

Solution

The activities are: A, B, C, D, E, F, G, H, I, and J

Determine the Sequence of the Activities

Some activities are dependent on the completion of others; a listing of the immediate predecessors/successors of each activity is useful for constructing the CPM network diagram.

Example 2

In example 1, all relationships are defined to be Finish-to-Start with Lag Time = 0 and the activity on node (AON) configuration is used. The following relationships are adopted:

A and B are the first activities

C and D are successors of A,

E and F are successors of B and F is one of the last activities,

G and H are successors of C,

I is a successor of G and is one of the last activities,

J is a successor of D, E, and H and is one of the last activities.

Describe the logic of this project.

Solution

A and B are the first activities

C succeeds A

D succeeds A

E succeeds B

F succeeds B and is one of the last activities

G succeeds C

H succeeds C

I succeeds G and is one of the last activities

J succeeds D, E, H and is one of the last activities

Draw the Network Diagram

A project network is a graph or a flow chart depicting the sequence in which a project's terminal elements are to be completed by showing terminal elements and their dependencies. Once activities and their sequencing have been defined, the CPM diagram can be drawn. CPM models the activities and events of a project as a network.

Example 3

Draw the network diagram given the precedence relationships among the activities as per example 2.

Solution

It is always advisable to start the project with a Start Milestone (Start) and end it with an End or Finish Milestone (End), as illustrated in Figure 2.
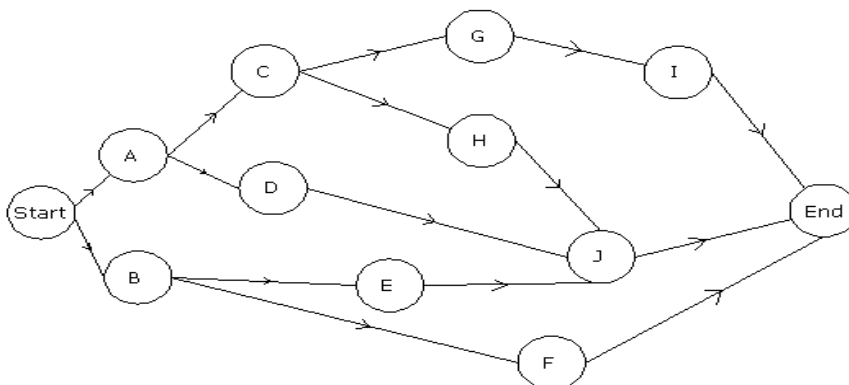

Figure 2 Network Diagram

Estimate the Completion Time for each Activity

The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number is used for an activity's time estimate. The activity completion time as normally calculated by dividing the original activity quantity Q as determined in the BOQ by the rate of productivity (ROP) based on a well-defined manpower. Normally, three rates of productivity are estimated for each activity, one being the most optimistic A, the second being the most pessimistic B, and the third being the most likely M; thus, three durations are estimated for each activity, one being the most optimistic duration (a = Q/A), the second being the most pessimistic duration (b = Q/B), and the third being the most likely duration (m = Q/M). The duration of each activity (D) is then calculated using the following equation:

$$D = \frac{a + b + 4m}{6}$$

Note that this duration is calculated based on a certain rate of productivity that is directly depending on the number of manpower. If the rate of productivity increases by increasing the number of manpower, then the activity duration decreases.

Example 4 (Activity Duration Computations)

Given the quantity (obtained from the BOQ), and the most optimistic rate of productivity (A), the most pessimistic rate of productivity (B), and the most likely rate of productivity (M) (according to a specified number of manpower per day) for each activity as shown in Table 1, calculate the duration of each activity.

Activity	Quantity (Q)	A	B	M
A	30	30	10	15
B	24	12	4	6
C	105	35	15	21
D	18	6	2	3
E	210	70	30	42
F	294	42	14	21
G	36	12	4	6
H	48	24	8	12
I	105	35	15	21
J	280	70	28	40

Table 1 Activities Quantity AND Rate of Productivity (most Optimistic, most Pessimistic, most Likely)

Solution

First of all, the most optimistic duration (a), the most pessimistic duration (b), and the most likely duration (m) of each activity are calculated, where the duration is equal to the quantity divided by the rate of productivity (a = Q/A, b = Q/B, m = Q/M). Then, the duration of each activity is calculated according to the following equation:

$$D = \frac{a + b + 4m}{6}$$

The duration in days of each activity is shown in Table 2.

Activity	a	b	m	Duration (D)
A	1	3	2	2
B	2	6	4	4
C	3	7	5	5
D	3	9	6	6
E	3	7	5	5
F	7	21	14	14
G	3	9	6	6
H	2	6	4	4
I	3	7	5	5
J	4	10	7	7

Table 2 Duration Calculation for each Activity

Note that the calculated duration for each activity as per Table 2, will be used throughout the following examples.

Identify the Critical Path

The critical path is the longest duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the following four parameters for each activity:

• ES – earliest start time: the earliest time at which the activity can start given that its precedent activities must be completed first.

•   EF – earliest finish time, equal to the earliest start time for the activity plus the time required to complete the activity.

•   LF – latest finish time: the latest time at which the activity can be completed without delaying the project.

•   LS – Latest start time, equal to the latest finish time minus the time required to complete the activity.

The slack time or float for an activity is the time between its earliest and latest start time, or between its earliest and latest finish time. Slack or float is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project.

The critical path is the path through the project network in which none of the activities have slack, that is, the path for which ES = LS and EF = LF for all the activities in the path. Any delay in the critical path delays the project. Similarly, to accelerate the project, it is necessary to reduce the total time required for the activities in the critical path.

The total float (TF) for an activity is defined to be the amount of time the ES of an activity could be delayed without delaying the entire project.

The free float (FF) for an activity is defined to be the amount of time the ES of an activity could be delayed without delaying the start of the immediate successors.

The interfering float (INTF) for an activity is defined to be the amount of time in which the completion of an activity may occur and not delay the termination of the project, but within which completion will delay the ES of some following activities.

The independent float (INDP) for an activity is defined to be the amount of time in which the completion of an activity may occur and not delay the termination of the project, not delay the early start of any following activities, and not be delayed by any preceding activities.

The roles of the total and free floats in scheduling non-critical activities are explained in terms of two general rules:

•   If the total float equals to the free float, the non-critical activity can be scheduled anywhere between its earliest start and latest completion time without delaying the project.

•   If the free float is less than the total float, the starting of the non-critical activity can be delayed relative to its earliest start time by no more than the amount of free float without delaying the start of the immediate succeeding activity nor delay the project.

Example 5 (Critical Path Calculations)

According to the calculated duration of each activity in example 4, determine the duration of the project and the critical path(s).

Solution

The activity icon that will be adopted is shown in Figure 3.
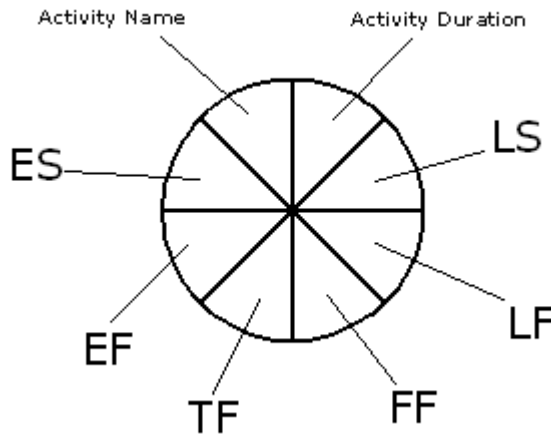
Figure 3 Activity Icon

EF = ES + D

LF = LS + D

TF = LS – ES = LF – EF

FF = minimum ES of succeeding activity – EF of activity

INTF = TF-FF

INDP = minimum ES of succeeding activity – maximum LF of preceding activity – D of activity

The start milestone (Start) is assigned an ES of zero, when the start of the project occurs at the first day. Then the EF of each activity is calculated as EF = ES + D.

For a F/S relationship with Lag = 0, when an activity has more than one predecessor, its ES is the maximum EF of the preceding activities since all predecessors must be completed before this activity starts. This procedure is known as the forward pass calculations. Once these calculations are completed, the ES and EF of each activity are known, and the project duration is also known, the EF of the end milestone determines the project duration which is also the

LF of the end milestone (End). The LS of the end milestone is calculated as LS=LF – D. The LF of each activity is the same as the LS of the succeeding activity. When an activity has more than one successor, its LF is the minimum of the LS of the succeeding activities. This procedure is known as the backward pass calculations. To verify the correctness of the computations, the LS which is the ES of the start milestone should be zero. The ES, EF, LS, LF, TF, and FF of each activity are provided in Figure 4.
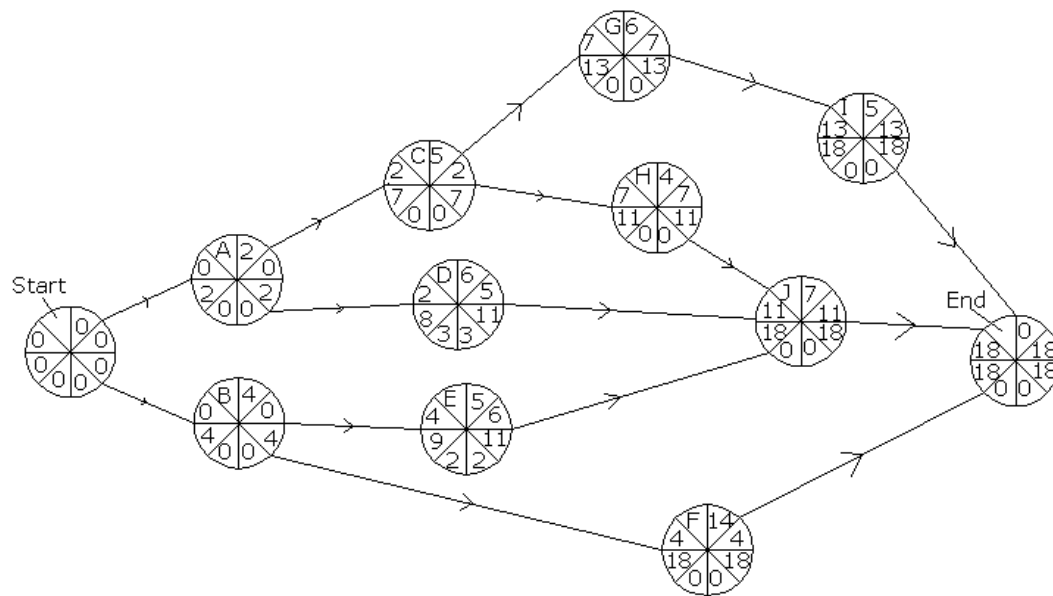
Figure 4 Network Diagram CPM Calculations

The importance of the CPM is that the non-critical activities may be delayed without affecting the duration of the whole project, in a way that the contractors can benefit time in another project.

The total float TF of an activity represents the amount of delay that the ES of an activity can tolerate without delaying the project.

The free float FF of an activity represents the amount of delay that the ES of an activity can tolerate without delaying the ES of the successor(s).

If the TF of an activity is = 0, then its FF = 0 and the activity is critical.

If the FF can be 0, this does not necessitate that the TF is = 0.

The amount of delay of an activity = Delay with respect to ES – TF of the activity.

If the amount of delay of an activity is less than or equal to 0, then the project is not delayed. If several activities are delayed, then compute the delay for each activity. If the activities are

on the same path, the total delay is the sum of all the individual delays. If the activities are on independent or parallel path(s), the total delay is the larger value.

It should be noted that the total project duration is 18 days, and there are three critical paths (TF of each activity on the critical path = 0):

Start – A – C – G – I – End

Start – A – C – H – J – End

Start – B – F – End

Discussion:

- *If activity B is delayed (ES of B) by 2 days, how will it affect the project and the succeeding activity(ies)?*

  TF of B is = 0, then the project will be delayed by 2 days (delay of B – TF of B).

  FF of B is = 0, then the ES of E will be delayed by 2 days (delay of B – FF of B), and the ES of F will also be delayed by 2 days (delay of B – FF of B).

- *If activity D is delayed by 2 days, how will this affect the project and the succeeding activity(ies)?*

  TF of D = 3, then the ES of D can be delayed by up to three days without delaying the project, so the project will still finish on time.

  FF of D = 3 (< 2), then the ES of J is not delayed.

- *If activity C is delayed by 5 days and activity H is delayed by 3 days, how will this affect the project and the succeeding activity (ies)?*

  TF of C = 0, then the project is delayed by 5 – 0 = 5 days, and TF of H = 0, then the project is delayed by 3 – 0 = 3 days; since C and H are on the same path, then the project is delayed by 5 + 3 = 8 days.

  FF of C = 0, then the ES of H is delayed by 5 – 0 = 5 days; and FF of H = 0, then the ES of J is delayed by 3 – 0 = 3 days.

- *If activity D is delayed by 5 days and activity E is delayed by 3 days, how will this affect the project and the succeeding activity (ies)?*

  TF of D = 3, then the project will be delayed by 5 – 3 = 2 days, and TF of E = 2, then the project is delayed by 3 – 2 = 1 days; and as D and E are not on the same path, then the project will be delayed by the maximum between 2 and 1, that is 2 days.

FF of D = 3, then the ES of J is delayed by 5 – 3 = 2 days; and FF of E = 2, then the ES of J is delayed by 3 – 2 = 1 day; then the ES of J is delayed by the larger value which is 2 days.

Schedule of Work (Gantt Charts)

A Gantt chart, commonly used in project management, is one of the most popular and useful ways of showing activities (tasks or events) displayed against time. On the left of the chart is a list of the activities and along the top is a suitable time scale. Each activity is represented by a bar; the position and length of the bar reflects the start date, duration and end date of the activity. This allows you to see at a glance:

•   What the various activities are

•   When each activity begins and ends

•   How long each activity is scheduled to last

•   Where activities overlap with other activities, and by how much

•   The start and end date of the whole project

The first Gantt chart was devised in the mid-1890s by Karol Adamiecki, a Polish engineer who ran a steel works in southern Poland and had become interested in management ideas and techniques.

Some 15 years after Adamiecki, Henry Gantt, an American engineer and project management consultant, devised his own version of the chart and it was this that became widely known and popular in western countries. Consequently, it was Henry Gantt whose name was to become associated with charts of this type.

Originally Gantt charts were prepared laboriously by hand; each time a project changed it was necessary to amend or redraw the chart and this limited their usefulness, continual change being a feature of most projects. Nowadays, however, with the advent of computers and project management software, Gantt charts can be created, updated and printed easily.

Today, Gantt charts are most commonly used for tracking project schedules. For this it is useful to be able to show additional information about the various tasks or phases of the project, for example how the tasks relate to each other, how far each task has progressed, what resources are being used for each task and so on.

Example 6

When is the project expected to finish assuming Sundays are off, if the project starts on Monday April 27, 2009 at 8:00 a.m.?

Solution

Referring to Figure 5, the last day at which the project ends is Saturday, May 16, 2009 at 5:00 p.m.
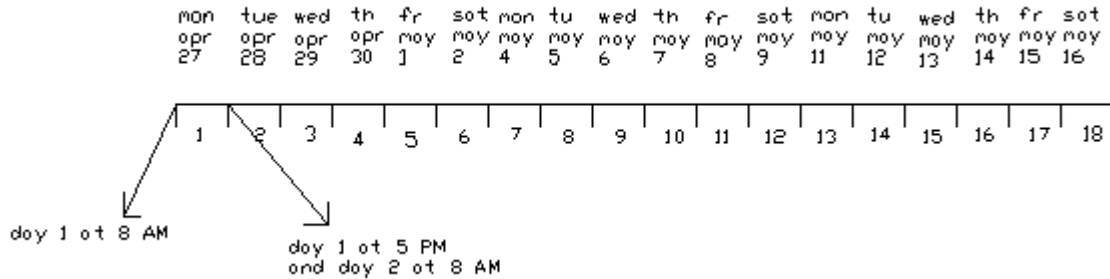
Figure 5 Illustration on the End Day of the Project

Example 7 (Project Scheduling)

Provide a schedule of work for the project.

Solution

The end product of network calculations is the construction of the schedule or time chart. It consists of scheduling an activity according to its ES, EF, LS, and LF. Figures 6 and 7 show the schedule of work of the project based on early start and late start, respectively.
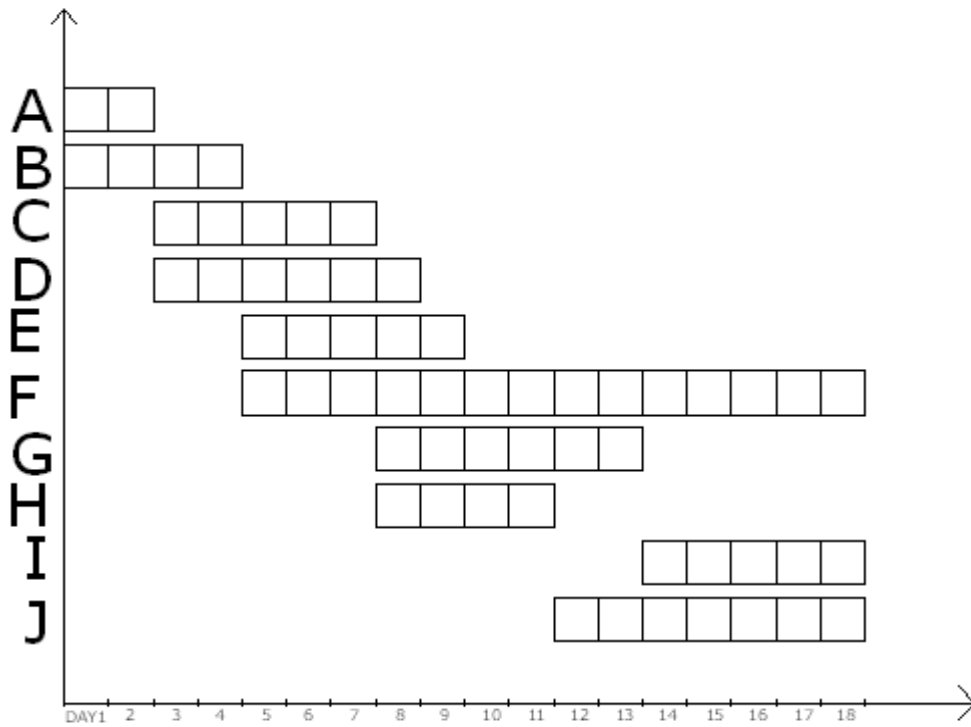
Figure 6 Schedule of Work based on Early Start
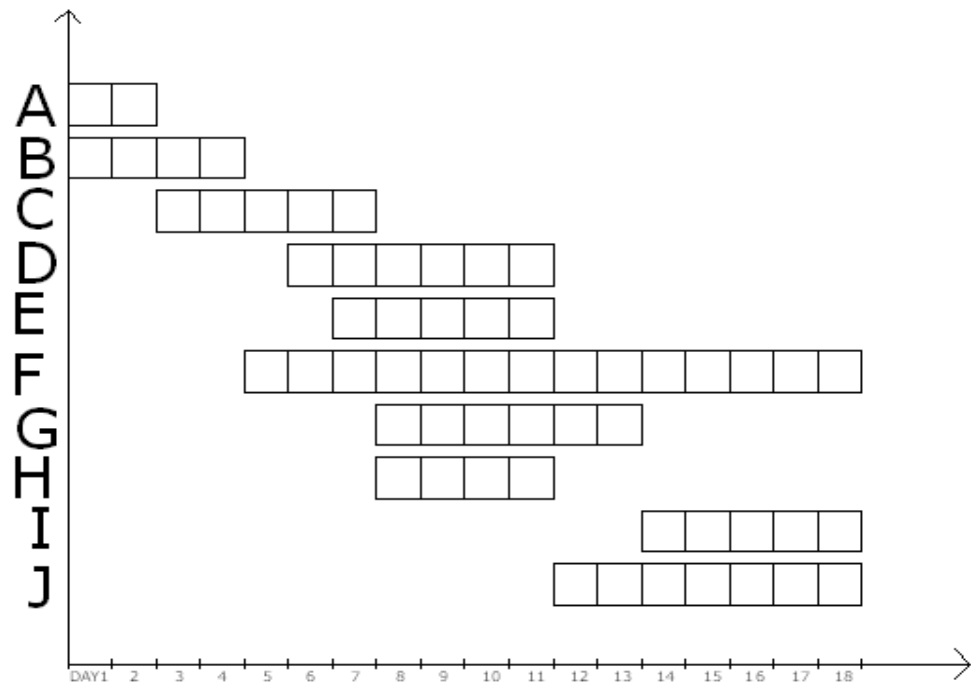
Figure 7 Schedule of Work based on Late Start

Schedule, Allocate, and Level the Resources when the Availability is Limited

The sequencing and scheduling of activities to accommodate limitations in the availability of resources is called resource allocating. Activities are analyzed to determine their scheduled level of resource requirement and match with the available resources. The project will either assign the required scheduled level to the activity to implement the project or will alter, in an efficient manner, the unacceptable work plan, into an implementable project.

In practice, however, a project will require the use of various resources such as equipment (tools and machines), manpower (professional and trade labors), and material. All of these resources can be presented in terms of project's financial requirements. Single resource scheduling will suffice for most projects; however, for complex projects or those requiring scarce and extraordinary resources, multi-resource scheduling must be considered.

Resource scheduling problems vary in kind and are usually a function of the nature of the project and its organizational setting. One category of resource scheduling problem is called "Resource Conflict Resolution" is when several concurrent activities compete for a fixed quantity of resource. The procedure for resolving the conflicts is to modify the unacceptable schedule to re-sequence activities such that all resource conflicts are resolved and the total project duration is increased by a minimum amount. Because of the nature of the procedure, it can be implemented only after the initial network has been laid out, the time schedule is determined, and the resource analysis is performed.

Another category of resource allocation problems is the time/cost tradeoff analysis or crashing analysis. This type of problem occurs when there are no constraints on the availability of resources; however, one may be interested in reducing the project completion time by accelerating certain activities at the expense of allocating more resources. It is important to note that time/cost tradeoff analysis and resource allocation techniques are two different procedures. The reason for including this type problem in this section is that as in the case of other resource allocation problems, the analysis also includes an implicit relationship between the cost of resources and the project duration.

The next category of problems, referred to as "Fixed Resource Limit Scheduling," arises when there are definite limitations on the amount of resource available to carry out the project. The scheduling objective in this case is to attempt to meet the original project due date subject to the fixed limits on resource availability; however, it is always possible that the project duration may increase beyond the initially established schedule.

The last category of problems, to be considered here, referred to as "Resource Leveling," occurs when a sufficient amount of resources is available to complete the project by a specified date; however, it is desirable or necessary to reduce the magnitude of variation in the utilization of a resource over the life of the project.

Example 8 (Cost Calculations)

For each activity, the following information is provided in Tables3, 4, and 5 (the first two columns for materials and the first three columns for manpower and machine), respectively:

Budgeted quantity for the material as per BOQ, unit cost for the material, number of manpower per day, unit rate for manpower per day, number of machine per day, unit rate for machine per day.

Determine the cost of materials, manpower, and machine for each activity, and the total cost of the project.

Solution

The total cost of each activity is the summation of the cost of material, the cost of manpower, and the cost of machine, i.e., the cost of the resources of the project. The total material cost of each activity is the product of the budgeted quantity and the unit cost of each material (column 2 times column 3). The total manpower for each activity is the product of the manpower per day, the daily rate of manpower, and the total number of days (column 2 times column 3 times column 4). The total machine cost for each activity is the product of the machine per day for each activity, the daily rate of each machine, and the total number of days (column 2 times column 3 times column 4).

Activity	Quantity for Material (L^2)	Unit Cost for Material($ /L^2$)	Total Material Cost ($)
A	30	140	4200
B	24	220	5280
C	105	17	1785
D	18	38	684
E	210	15	3150
F	294	14	4116
G	36	6	216
H	48	8	384
I	105	53	5565
J	280	38	10640

$$\sum = 36020 \ \$$$

Table 3 Material Cost Calculations

Activity	Manpower/ Day	Unit Cost/ Manpower/Day	Duration in Days	Total Manpower Cost ($)
A	15	20	2	600
B	4	20	4	320
C	8	20	5	800
D	6	20	6	720
E	5	20	5	500
F	5	20	14	1400
G	4	20	6	480
H	5	20	4	400

I	10	20	5	1000
J	5	20	7	700

$$\sum = 6920 \, \$$$

Table 4 Manpower Cost Calculations

Activity	Machine/ Day	Unit Cost/ Machine/Day	Duration in Days	Total Machine Cost ($)
A	1	100	2	200
B	1	120	4	480
C	1	110	5	550
D	1	50	6	300
E	1	70	5	350
F	1	80	14	1120
G	1	30	6	180
H	1	20	4	80
I	1	10	5	50
J	1	40	7	280

$$\sum = 3590 \, \$$$

Table 5 Machine Cost Calculations

Now, the final total cost of the project is calculated by adding the cost of all the resources for each activity, as shown in Table 6.

ACTIVITY	MATERIAL	MANPOWER	MACHINE	TOTAL COST
A	4200	600	200	5000
B	5280	320	480	6080
C	1785	800	550	3135
D	684	720	300	1704
E	3150	500	350	4000
F	4116	1400	1120	6636
G	216	480	180	876
H	384	400	80	864
I	5565	1000	50	6615
J	10640	700	280	11620
TOTAL	36,020$	6,920$	3,590$	**46,530$**

Table 6 Project Cost Calculations

The total cost of the project is:

36020$+6920$+3590$ = 46530 $ which represents the total direct costs of the project.

Note that this is not the bidding cost of the project as the indirect costs (overhead costs and contingency costs) should be added to the direct costs.

Example 9 (Histogram and S-Curve)

Draw the histogram and the S-curve of the project.

Solution

The histogram represents the scheduled cost per day throughout the duration of the project. It could be done based on ES or LS. Unless otherwise specified, the total cost of each activity is

divided by its duration and this results in the daily budgeted cost of each activity. Next, allocate the daily cost of each activity referring to the already developed schedule and last, sum up on a daily basis, all the costs per day that are expected, as illustrated in Figures 8 and 9, based on ES and LS, respectively.

The S-curve represents the total cost that has occurred on a specified date. Note that, in the S-curve (based on ES or LS), the cost at the last day of the project (day 18) represents the total cost of the project (46,530$), as illustrated in Figure 10, based on ES and LS.
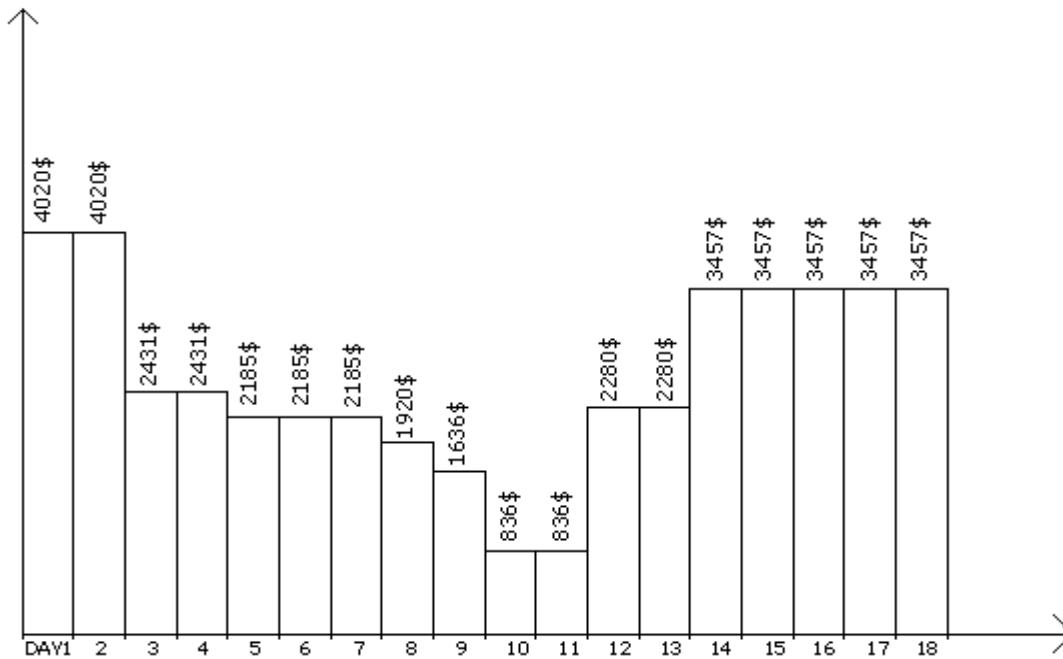
Histogram based on ES, 1 horizontal unit = 1 day, 1 vertical unit = 500$

Figure 8 Histogram of the Project Based on ES

Note that this study is only true when the material, manpower, and machine's total costs are equally divided on the number of days along which this activity is projecting.
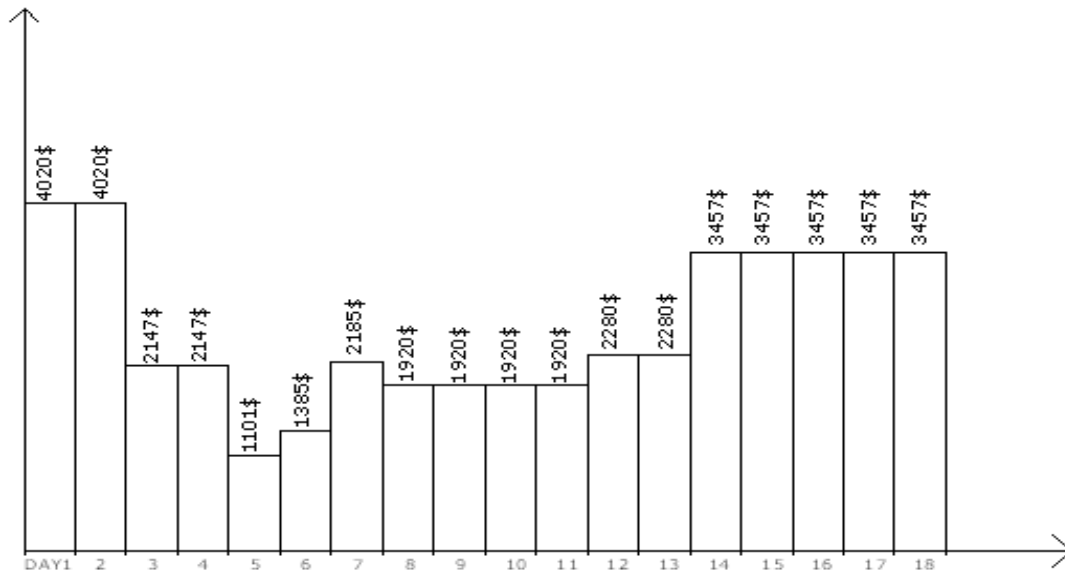
Histogram based on ES, 1 horizontal unit = 1 day, 1 vertical unit = 500$

Figure 9 Histogram of the Project Based on LS
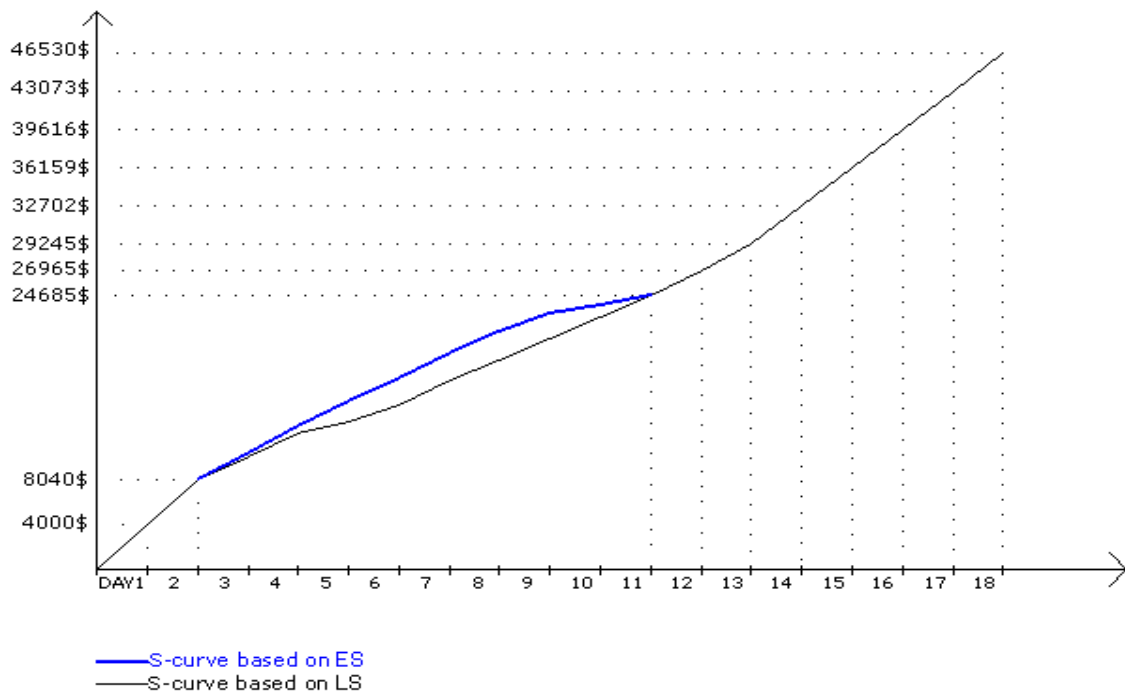
Figure 10 S-Curve of the Project Based on ES and LS

Example 10 (Histogram and S-Curve with Varying Daily Cost)

Assume that the material cost is paid on the last day of the activity, the machine cost is paid on the first day of the activity, and the manpower cost is paid on a daily basis (as before), then what would be the histogram and the S-curve of this distribution look like?

Solution

- Activity A:

Material:	0	4200
Manpower:	300	300
Machines:	200	0

500	4500

- Activity B:

Material:	0	0	0	5280
Manpower:	80	80	80	80
Machines:	480	0	0	0

560	80	80	5360

- Activity C:

Material:	0	0	0	1785
Manpower:	160	160	160	160
Machines:	550	0	0	0

710	160	160	1945

The same procedure is performed for all the other activities. The histogram and S-curve based on ES and LS are shown in Figures 11, 12, and 13, respectively
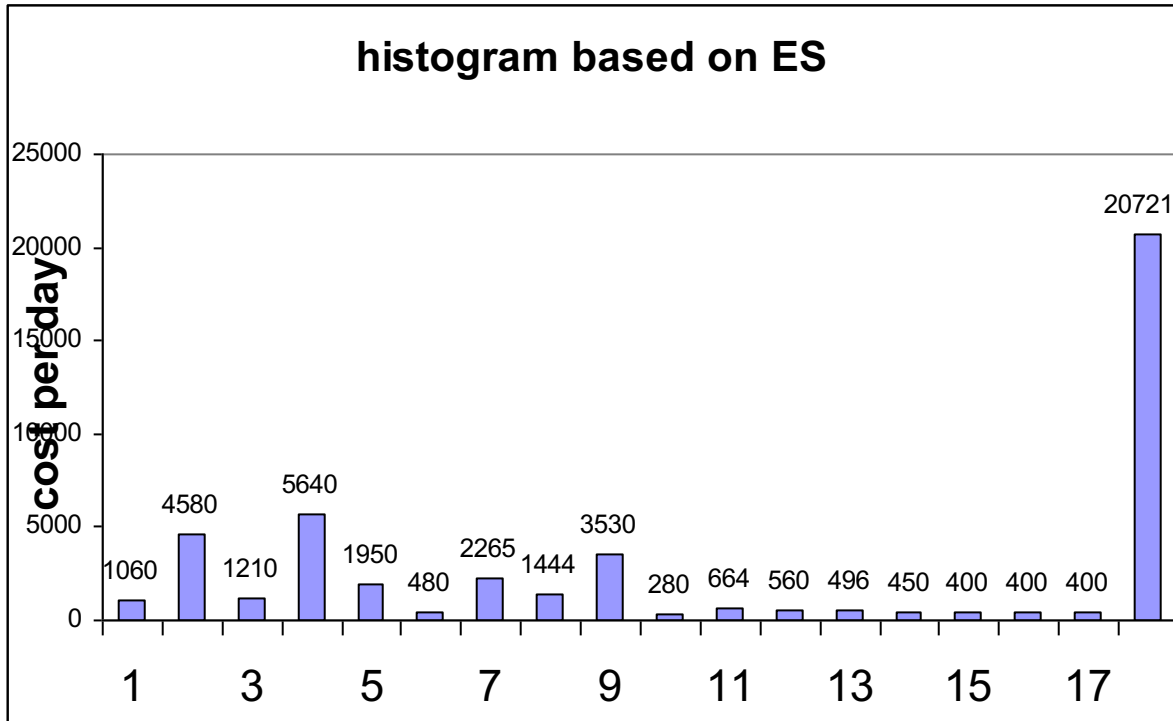
Figure 11 Histogram of the Project Based on ES with Varying Daily Costs
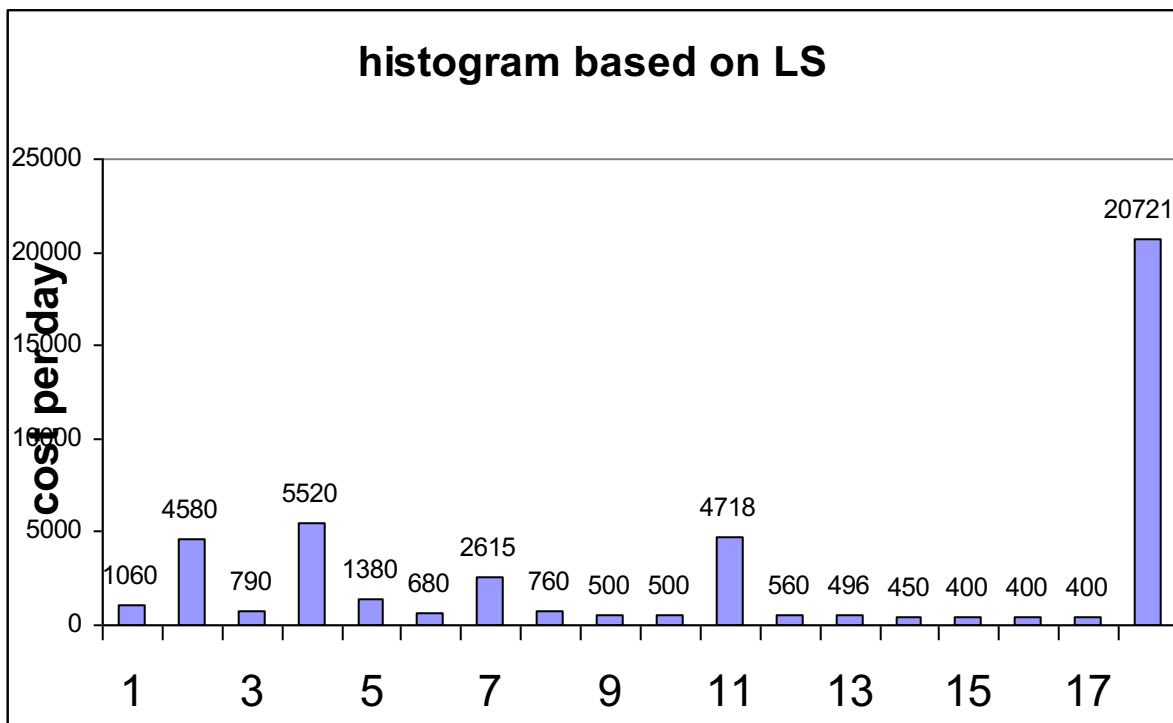
Figure 12 Histogram of the Project Based on LS with Varying Daily Costs
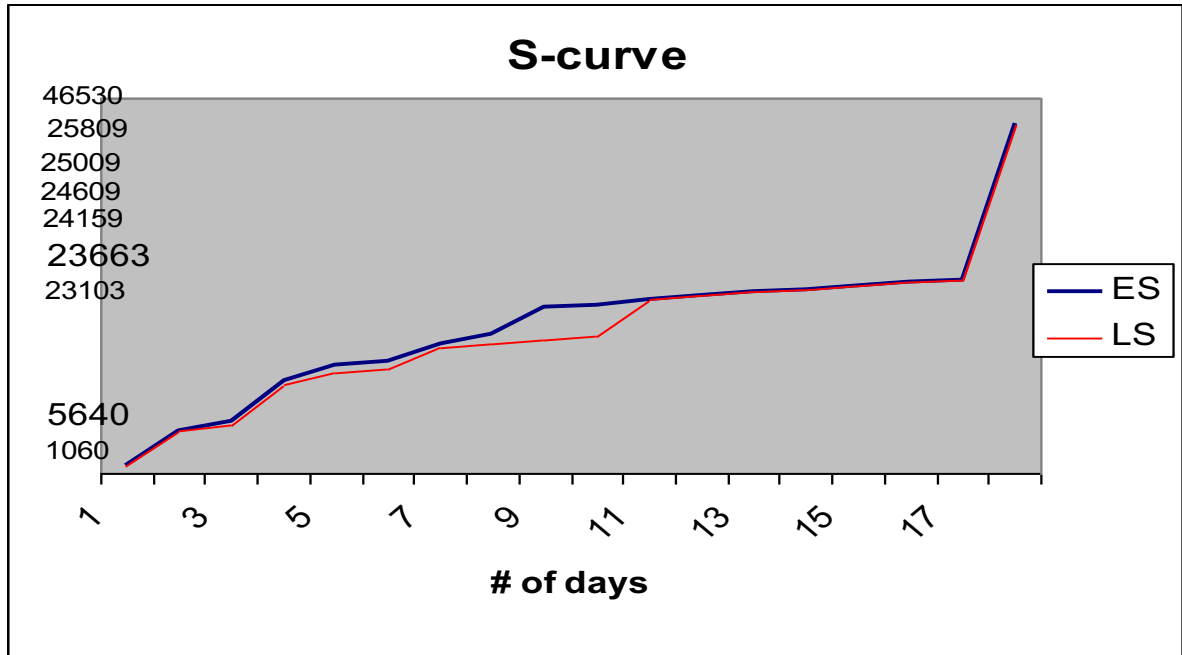
Figure 13 S-Curve of the Project Based on ES and LS with Varying Daily Costs

It should also be noted that, regardless how the resources are paid, in the S-curve (based on ES or LS), the cost at the last day of the project (day 18) represents the total cost of the project (46,530$).

Example 11 (Daily Manpower Scheduling)

Determine the number of manpower that is needed per day for the project.

Solution

The same procedure that is followed in the above example is adopted, except that the cost per day for each activity is substituted with the allocated manpower per day for each activity. The number of manpower that is needed per day based on ES and LS is shown in Figures 14 and 15, respectively.

Figure 14 Manpower per Day Based on ES
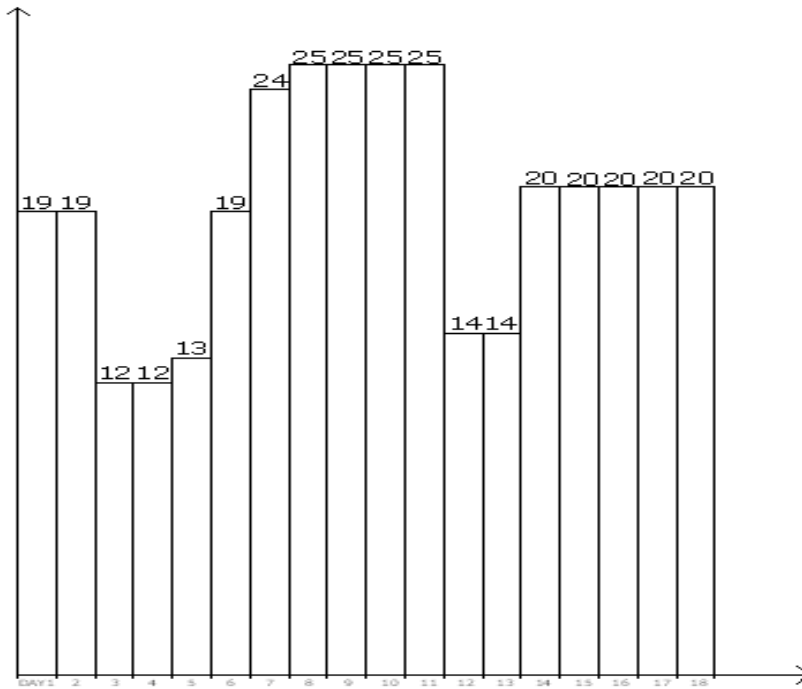
Figure 15 Manpower per Day Based on LS

Example 12 (Daily Manpower Scheduling with Varying Categories)

Now, the above-described procedure can be performed if the category of manpower is the same for all activities, but what if the category of manpower varies among the activities? Assume the category of manpower is distributed as shown in Table 7.

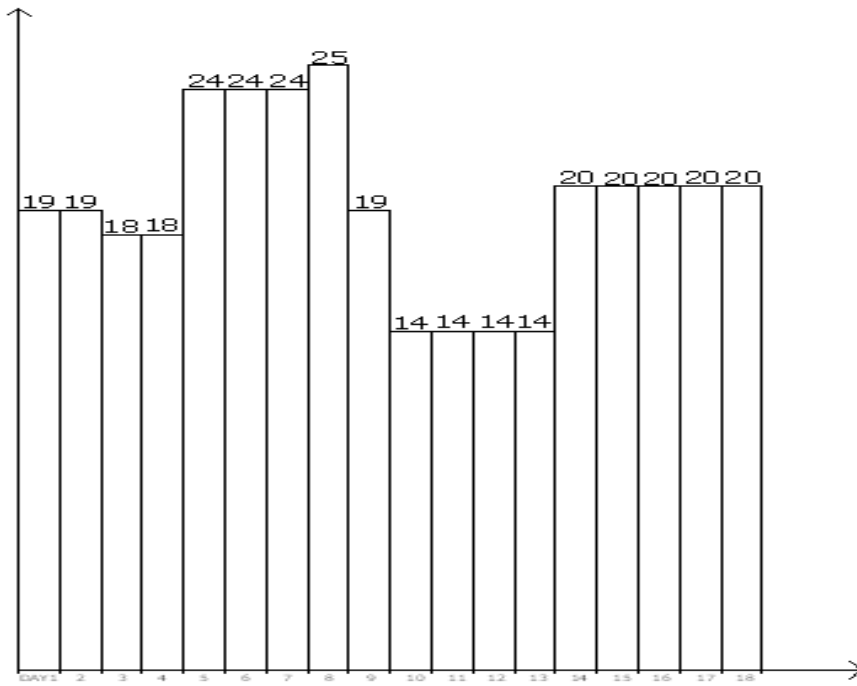| ACTIVITY	MANPOWER
A	15/X
B	4/Y
C	8/Z
D	6/X
E	5/X
F	5/Z
G	4/Y
H	5/Y
I	10/X
J	5/Z

Table 7 Manpower Number and Category for each Activity

Where, X is the category of manpower for activities A, D, E, and I; Y is the category of manpower for activities B, G, and H; Z is the category of manpower for activities C, F, and J. The manpower for each activity is assigned on a daily basis; Table 8 shows the values of manpower that is needed per day for ES and LS, respectively, following the same procedure as per the above example.

Solution

Day	Number of Manpower based on ES	Number of Manpower based on LS
1	15X+4Y	15X+4Y
2	15X+4Y	15X+4Y
3	4Y+8Z+6X	4Y+8Z
4	4Y+8Z+6X	4Y+8Z
5	13Z+11X	13Z
6	13Z+11X	13Z+6X
7	13Z+11X	13Z+11X
8	9Y+11X+5Z	11X+5Z+9Y

9	9Y+11X+5Z	11X+5Z+9Y	
10	5Z+9Y	11X+5Z+9Y	
11	5Z+9Y	11X+5Z+9Y	
12	10Z+4Y	10Z+4Y	
13	10Z+4Y	10Z+4Y	
14	10Z+10X	10Z+10X	
15	10Z+10X	10Z+10X	
16	10Z+10X	10Z+10X	
17	10Z+10X	10Z+10X	
18	10Z+10X	10Z+10X	

Table 8 Manpower per Day Based on ES and LS, with Varying Category per Activity

Example 13 (Resource Allocation)

Activities C, D, and E need a crane each, and only two cranes are available. Describe how the two cranes shall be distributed among the three activities.

Solution

Activities C and D have the same ES (day 2), whereas activity E has an ES of 4. The two cranes shall be allocated to activities C and D, and then the activity that finishes first will provide its crane to activity E. So activity E may immediately succeed either activity C or activity D. if activity E succeeds activity C, the increase in project duration is IPDCE = EFC – LSE = 7 – 6 = 1 day. If activity E succeeds activity D, the increase in project duration is IPDDE = EFD – LSE = 8 – 6 = 2 days. Due to the fact that there is a limited number of cranes (2 cranes), and activities C, D, and E require a crane each, allocate the two cranes to activities C and D, then once activity C finishes, allocate its crane to activity E, which results in a project delay of 1 day. Note that a negative IPD means that the project will not be delayed.

Example 14 (Resource Leveling)

Provide a schedule taking into consideration that the maximum available daily manpower is limited to 20.

Solution

Referring to Example 11, where a schedule has been provided for the project showing the daily manpower, it is observed that certain days require that the daily manpower exceeds 20. In order that the maximum daily manpower is limited to 20, specific activities need to be shifted (keeping the same logic of the project). It is always advisable to start shifting non-critical activities with larger total float, which may result in a lesser increase in project duration. Additionally, if the activities that may be shifted have the same total float (or if they are critical activities), shifting shall be exercised on the activities with lesser number of manpower per day.

To level the given project based on ES, proceed as follows and illustrated in Figure 16:

- The first four days satisfy the 20 manpower per day constraint.

- Day 5 violates the criteria (24 manpower), so shifting E by 1 day reduces the manpower to $24 – 5 = 19$ and will not delay the project; now, the manpower at day 10 is $14 + 5 = 19$. It should be noted that shifting activity F by 1 day also satisfies the criteria but results in increasing the project duration by 1 day, that is why it was not selected.

- Day 6 violates the criteria (24 manpower), so shift E by 1 day reduces the manpower to $24 – 5 = 19$ and will not delay the project; now, the manpower at day 11 is $14 + 5 = 19$.

- Day 7 violates the criteria (24 manpower), so shift E by 1 day reduces the manpower to $24 – 5 = 19$; however, J (successor of E) cannot start before E finishes, so adjust by shifting J by 1 day, which results in delaying the project by 1 day; now, the manpower at day 12 is $14 + 5 = 19$ and the manpower at day 19 is 5.

- Day 8 violates the criteria (25 manpower), so shift H (not critical any more) by 1 day reduces the manpower to $25 – 5 = 20$. It should be noted that shifting E instead also satisfies the criteria but results in delaying the project by 1 day as E became a critical activity in Step 4.

As observed, the project will be delayed by 1 day. Note that predecessor/successor relationship has been respected.
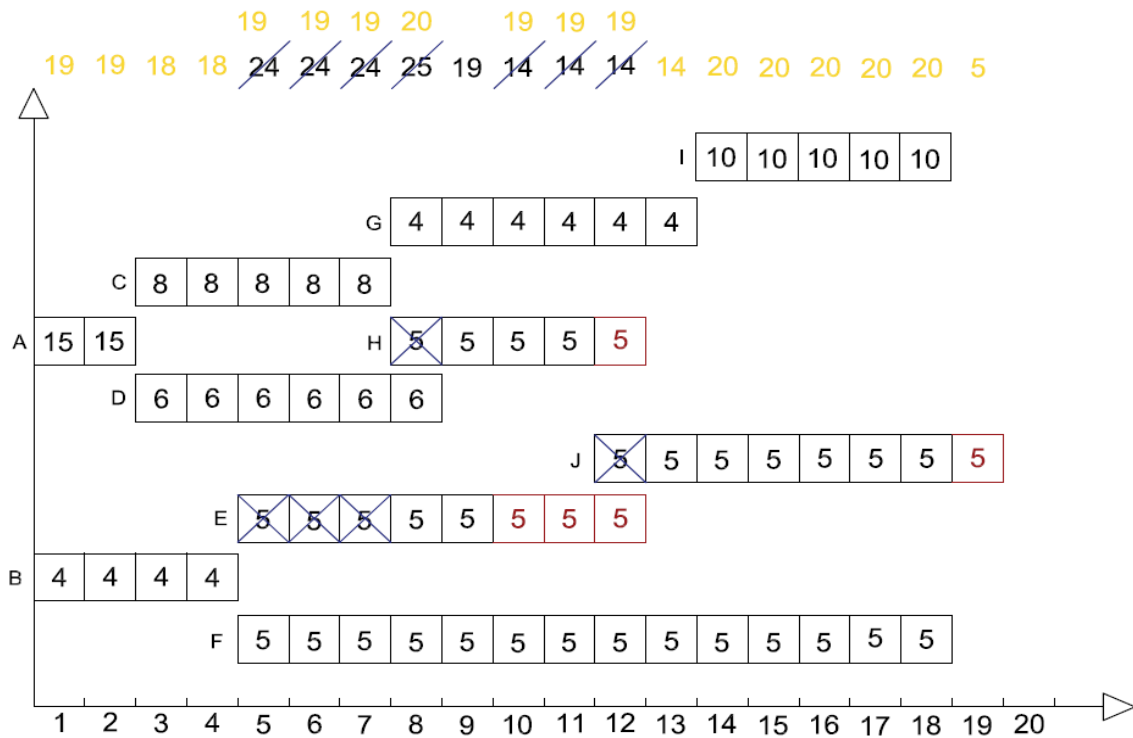
Update the CPM Diagram as the Project Progresses

Example 15 (Project Updating)

An update is done on day 14, and the actual quantities for each activity are reported as follows in Table 9. What can you conclude about the progress of work?

Activity	Budgeted Quantity	Actual Quantity at Day 14
A	30	30
B	24	24
C	105	105
D	18	18

E	210	126
F	294	147
G	36	18
H	48	24
I	105	0
J	280	0

Table 9 Quantity Update at Day 14

Solution

It is necessary at this stage to define the percent completion for each activity which is the ratio of the actual quantity and the budgeted quantity. Then, the remaining duration (RD) for each activity is simply the original duration (OD) times the percent that has not been completed (100% - %complete). The percent complete and the remaining duration for each activity are shown in Table 10.

Activity	Percent Complete	RD
A	100	0
B	100	0
C	100	0
D	100	0
E	60	2
F	50	7
G	50	3
H	50	2
I	0	5
J	0	7

Table 10 Percent Complete and Remaining Duration

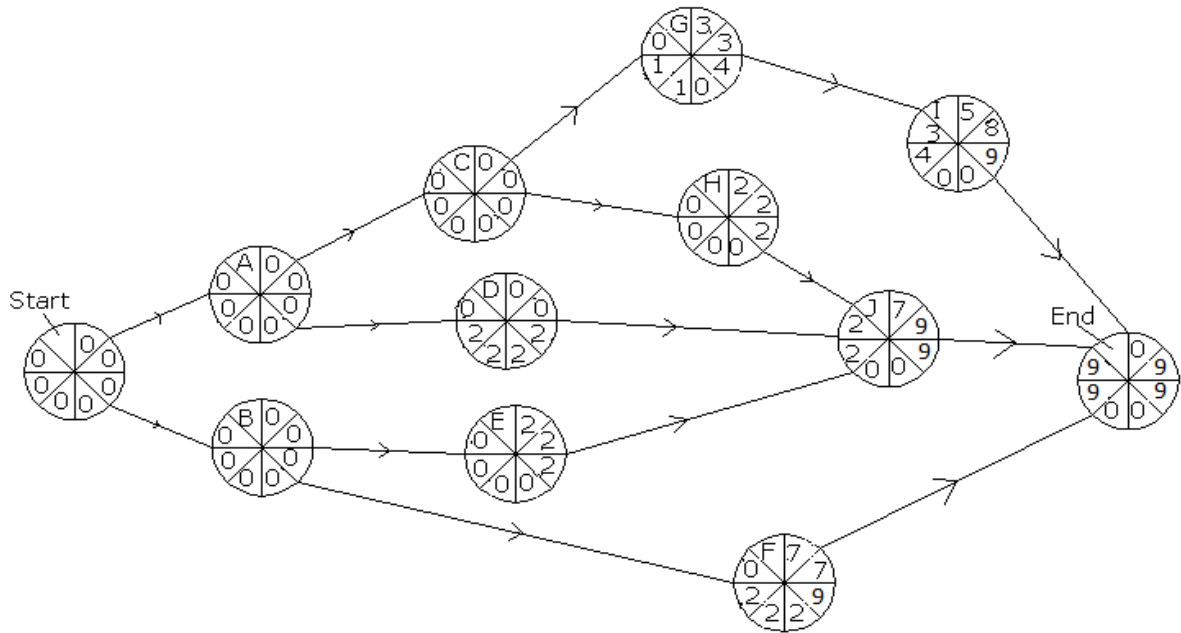Next, the new project duration is calculated as shown in Figure 17.

Figure 17 Project Duration Calculations based on Project Updating

The new project duration is 9 days, to be added to the day when the update was performed (day 14), and according to this progress of work, the project will finish in $14 + 9 = 23$ days, which is larger than the original project duration (18 days), meaning that the project is behind schedule. Note that the critical paths have changed from the scheduling part, and they are: Start-A-C-H-J-End and Start-B-E-J-End.

Example 16 (Project Updating, ct'd)

Activity F was supposed to be 75% complete today, but it is 50% complete with actual cost for now of $6000. Discuss the status of activity F.

Answer

The cost variance (CV) is defined to be the difference between the budgeted cost of work performed (BCWP = % Complete x Budgeted Cost) and the actual cost of work performed (ACWP). For activity F, the CV = (50%) (6636) – 6000 = - $2682 < 0

Additionally, the scheduled variance (SV), is defined to be the difference between the budgeted cost of work performed and the budgeted cost of work scheduled (BCWS). For activity F, the SV = (50%) (6636) – (75%) (6636) = -$1659 < 0

As a conclusion, for activity F, there is an overrun in the expenditures (CV < 0) and the activity is behind schedule (SV < 0).

Control the CPM Diagram Assuming fast-Track Project

As the resources are increased, the time required to complete an activity decreases with a corresponding increase in cost, until a duration is reached below which is physically impossible to complete the activity, regardless of the level, quality, and/or cost of the resources employed. This duration is called the crash duration that is associated with a crash cost. It is important to define the slope of an activity which is defined as the absolute value of the ratio of (the difference between the normal cost and the crash cost) and (the difference between the normal duration and the crash duration). The slope of an activity is defined to be the daily cost increase for a duration reduction of one day.

Generally, the critical activity that has the smallest slope (i.e. lowest cost increase) is selected for a one-day reduction of the project duration. If there are several critical paths, the activities with the smallest slopes for each path are selected, summed up, and compared with the slopes of critical activities that are common to these critical paths. The selection that results in the smallest daily cost increase is adopted. It is important to notice that in the crashing procedure, the original critical path shall be maintained, in addition to newly developed critical paths that have resulted from the crashing procedure. The crashing procedure stops when the original critical path(s) can no longer be crashed, or when a newly developed critical path can no longer be crashed.

Example 17 (Project Controlling assuming Fast-Track Project)

Given the crash durations and the crash cost for each activity as shown in Table 11, determine the new project duration and project cost under crash conditions, in the scheduling phase, i.e. before construction has started (Project considered as Fast-Track).

Solution

The slope of each activity is calculated as shown in Table 11.

Activity	Normal Cost	Normal Duration	Crash Cost	Crash Duration	Slope
A	5000	2	10000	1	5000
B	6080	4	12000	2	2960
C	3135	5	6000	2	955
D	1704	6	4000	4	1148

E	4000	5	8000	3	2000
F	6636	14	14000	10	1841
G	876	6	1600	4	362
H	864	4	2600	2	868
I	6615	5	14000	3	3692.50
J	11620	7	24000	3	3095

Table 11 Project Controlling for Fast-Track Project

The three critical paths are:

Start – A – C – G – I – End with duration of 18 days.

Start – A – C – H – J – End with duration of 18 days.

Start – B – F – End with duration of 18 days.

The other non-critical paths are:

Start – A – D – J – End with duration of 15 days.

Start – B – E – J – End with duration of 16 days.

The smallest slope of Start – A – C – G – I – End is 362$/day (activity G)

The smallest slope of Start – A – C – H – J – End is 868$/day (activity H)

The smallest slope of Start – B – F – End is 1841$/day (activity F)

The sum of the three slopes equals to 3071$/day which represents the additional cost that will result from reducing the total project duration to 17 days. However, since the first path and the second path have two common activities (A and C), where the slope of C is 955$/day, adding this slope to the slope of F (1841$/day) will result in a cost increase of 2796$/day which is more beneficial than the first option (3071$/day). The procedure will continue in the same manner until the original critical paths and the newly developed paths can no longer be crashed. In brief, the following represents the complete crashing procedure:

• Crash C + F by 1 Day, add 955$ + 1841$. Original critical paths remain the same with 17 days duration.

• As no new critical path(s) has developed, nor an activity can no longer be crashed from the step above, repeat step 1, i.e., Crash C + F by 1 Day add 955$ + 1841$. Original critical paths remain the same with 16 days duration and a new critical path has developed Start – B – E – J – End.

- Crash C + B by 1 Day, add 955$ + 2960$. Original critical paths with Start – B – E – J – End remain the same with 15 days duration and a new critical path has developed Start – A – D – J – End.

C can no longer be crashed.

The other suggestions are: min (G, I), + F + E = 362 + 868 + 1841 + 2000 = 5071$, and A + B = 5000 + 2960 = 7960$, and min (G, I) + J + F = 362 + 3095 + 1841 = 5298$, C + E + F = 955 + 2000 + 1841 = 4796$

- Crash min (G, I) + F + J by 1 Day, add 362$ + 1841$ + 3095$. All paths are critical with 14 days duration.

The other suggestions are: A + B = 5000 + 2960 = 7960$, and min (G, I) + H + F + D + E= 362 + 868 + 1841 + 1148 + 2000 = 6219$, A + F + E = 5000 + 1841 + 2000 = 8841$, G + H + D + B = 362 + 868 + 1148 + 2960 = 5338$

- As no new critical path(s) has developed, nor an activity can no longer be crashed from the step above, repeat above step, i.e., Crash G + F + J by 1 Day add 362$ + 1841$ + 3095$. All paths are critical with 13 days duration.

G and F can no longer be crashed.

- Crash A and B by 1 Day, add 5000$ + 2960$. All paths are critical with 12 days duration.

A and B can no longer be crashed.

Since B and F can no longer be crashed, the critical path Start – B – F – End can no longer be crashed and the crashing procedure stops at this point with a decrease in project duration of 6 days at an additional cost of 28063$.

New project duration after total crashing is 18 – 6 = 12 Days.

New project cost after total crashing is original cost + crash cost = 46530$ + 28063$ = 74593$.

Control the CPM Diagram after Updating

Example 18 (Project Controlling after Updating)

Crash the project during construction, as a result of the updating procedure in example 16.

Solution

First remove all activities that are 100% complete, which are in this case: A, B, C and D. Then, multiply the crashed duration of each activity by (100% - % complete) of that activity as shown in Table 12.

Activity	Normal Cost	RD	Crash Cost	Crash Duration	% Complete	Crash Duration	Slope
E	4000	2	8000	3	60	1	2000
F	6636	7	14000	10	50	5	1841
G	876	3	1600	4	50	2	362
H	864	2	2600	2	50	1	868
I	6615	5	14000	3	0	3	3692.50
J	11620	7	24000	3	0	3	3095

Table 12 Project Controlling as a Result of Updating

The two critical paths are:

Start – A – C – H – J – End with duration of 9 days.

Start – B – E – J – End with duration of 9 days.

The other non-critical paths are:

Start – A – C – G – I – End with duration of 8 days.

Start – A – D – J – End with duration of 7 days.

Start – B – F – End with duration of 7 days.

The smallest slope of Start – A – C – H – J – End is 868$/day (activity H)

The smallest slope of Start – B – E – J – End is 2000$/day (activity E)

The sum of the two slopes equals to 2868$/day which represents the additional cost that will result from reducing the total project duration to 8 days. The procedure will continue in the same manner until the original critical paths and the newly developed paths can no longer be crashed. In brief, the following represents the complete crashing procedure:

- Crash H and E by 1 Day, add 868$ + 2000$. Original critical paths remain the same with 8 days duration and a new critical path has developed Start – A – C – G – I – End.

- E and H can no longer be crashed.

- Crash J and G by 1 Day, add 3095\$ + 362\$. Critical paths remain the same with 7 days duration and two new critical paths have developed Start – B – F – End and Start – A – D – J – End.

G can no longer be crashed.

- Crash J, I and F by 1 Day, add 3095\$ + 3692.5\$ + 1841\$. Critical paths remain the same with 6 days duration.

C can no longer be crashed.

- Crash J, I and F by 1 Day, add 3095\$ + 3692.5\$ + 1841\$. Critical paths remain the same with 5 days duration.

I and F can no longer be crashed.

Since B and F can no longer be crashed, the newly developed critical path Start – B – F – End can no longer be crashed and the crashing procedure stops at this point with a decrease in project duration of 4 days at an additional cost of 23582\$.

New project duration after total crashing is $14 + 9 - 4 = 19$ Days (Delay of 1 day).

New project cost after total crashing is original cost + crash cost = 46530\$ + 23582\$ = 70112\$.

Section 5 — References

- Precedence and Arrow Networking Techniques for Construction by R.B. Harris, John Wiley and Sons.

- Project Management: A Systems Approach to Planning, Scheduling, and Controlling by H. Kerzner, Van Norstrand Reinhold, 10th Edition.

- Introduction to Operations Research, Hiller and Lieberman, McGraw Hill, 8th Edition.